Chapter

0

Vectors

Introduction of Vector

Physical quantities having magnitude, direction and obeying laws of vector algebra are called vectors.

Example : Displacement, velocity, acceleration, momentum, force, impulse, weight, thrust, torque, angular momentum, angular velocity *etc.*

If a physical quantity has magnitude and direction both, then it does not always imply that it is a vector. For it to be a vector the third condition of obeying laws of vector algebra has to be satisfied.

Example : The physical quantity current has both magnitude and direction but is still a scalar as it disobeys the laws of vector algebra.

Types of Vector

(1) **Equal vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(2) **Parallel vector :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(i) Both have same direction.

(ii) One vector is scalar (positive) non-zero multiple of another vector.

(3) **Anti-parallel vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when

(i) Both have opposite direction.

(ii) One vector is scalar non-zero negative multiple of another vector.

(4) **Collinear vectors :** When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear.

(5) **Zero vector** $(\vec{0})$**:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

(6) **Unit vector :** A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap or A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

(7) **Orthogonal unit vectors** $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb). The

$$\hat{i} = \frac{\vec{x}}{x}, \hat{j} = \frac{\vec{y}}{y}, \hat{k} = \frac{\vec{z}}{z}$$

$$\therefore \ \vec{x} = x\hat{i}, \ \vec{y} = y\hat{j}, \ \vec{z} = z\hat{k}$$

(8) **Polar vectors :** These have starting point or point of application . Example displacement and force *etc.*

(9) **Axial Vectors :** These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.

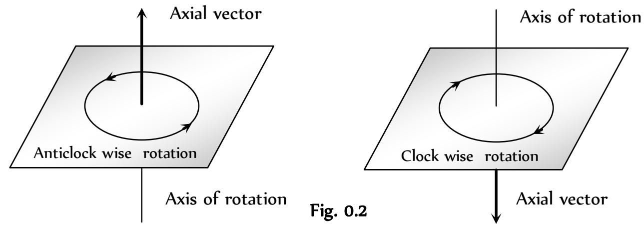


Fig. 0.2

(10) **Coplanar vector** : Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

Triangle Law of Vector Addition of Two Vectors

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. *i.e.* $\vec{R} = \vec{A} + \vec{B}$

$$\because \ \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

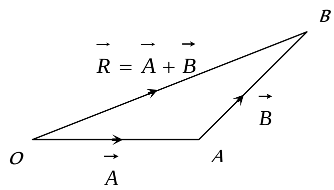


Fig. 0.3

(1) **Magnitude of resultant vector**

In $\Delta\, ABN$, $\cos\theta = \dfrac{AN}{B}$ $\therefore\ AN = B\cos\theta$

$\sin\theta = \dfrac{BN}{B}$ $\therefore\ BN = B\sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

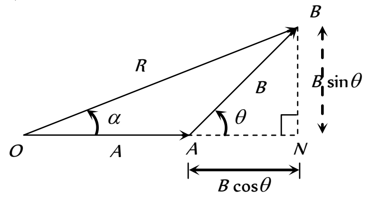


Fig. 0.4

$$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

(2) **Direction of resultant vectors :** If θ is angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Parallelogram Law of Vector Addition

If two non zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(1) **Magnitude**

Since, $R^2 = ON^2 + CN^2$

$$\Rightarrow R^2 = (OA + AN)^2 + CN^2$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\therefore\quad R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

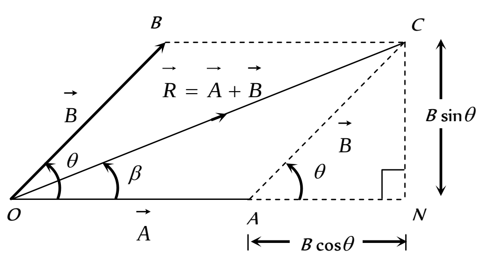


Fig. 0.5

Special cases : $R = A + B$ when $\theta = 0°$

$R = A - B$ when $\theta = 180°$

$R = \sqrt{A^2 + B^2}$ when $\theta = 90°$

(2) **Direction**

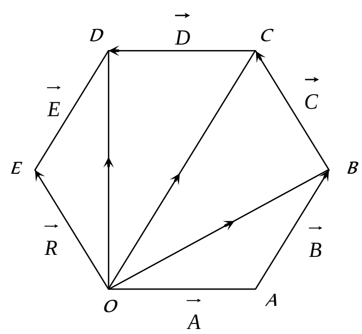
$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

Polygon Law of Vector Addition

If a number of non zero vectors are represented by the $(n-1)$ sides of an n-sided polygon then the resultant is given by the closing side or the n side of the polygon taken in opposite order. So,

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$$



Note :
- Resultant of two unequal vectors can not be zero.
- Resultant of three co-planar vectors may or may not be zero
- Resultant of three non co- planar vectors can not be zero.

Fig. 0.6

Subtraction of vectors

Since, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^o - \theta)}$$

Since, $\cos(180 - \theta) = -\cos\theta$

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

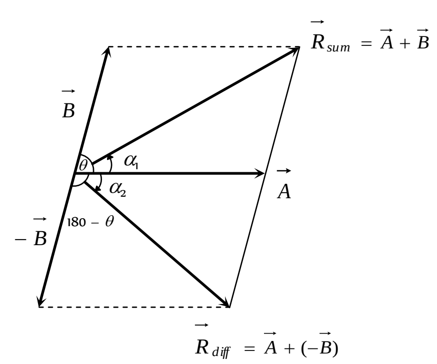


Fig. 0.7

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta}$$

and $\tan\alpha_2 = \dfrac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$

But $\sin(180-\theta)=\sin\theta$ and $\cos(180-\theta)=-\cos\theta$

$$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A-B\cos\theta}$$

Resolution of Vector Into Components

Consider a vector $\vec{R}$ in X-Y plane as shown in fig. If we draw orthogonal vectors $\vec{R}_x$ and $\vec{R}_y$ along x and y axes respectively, by law of vector addition, $\vec{R}=\vec{R}_x+\vec{R}_y$

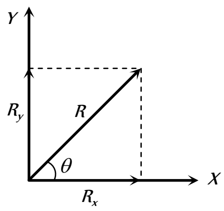


Fig. 0.8

Now as for any vector $\vec{A}=A\hat{n}$ so, $\vec{R}_x=\hat{i}R_x$ and $\vec{R}_y=\hat{j}R_y$

so $\vec{R}=\hat{i}R_x+\hat{j}R_y$ ...(i)

But from figure $R_x=R\cos\theta$ ...(ii)

and $R_y=R\sin\theta$ ...(iii)

Since R and θ are usually known, Equation (ii) and (iii) give the magnitude of the components of $\vec{R}$ along x and y-axes respectively.

Here it is worthy to note once a vector is resolved into its components, the components themselves can be used to specify the vector as

(1) The magnitude of the vector $\vec{R}$ is obtained by squaring and adding equation (ii) and (iii), *i.e.*

$$R=\sqrt{R_x^2+R_y^2}$$

(2) The direction of the vector $\vec{R}$ is obtained by dividing equation (iii) by (ii), *i.e.*

$$\tan\theta=(R_y/R_x) \text{ or } \theta=\tan^{-1}(R_y/R_x)$$

Rectangular Components of 3-D Vector

$$\vec{R}=\vec{R}_x+\vec{R}_y+\vec{R}_z q \text{ or } \vec{R}=R_x\hat{i}+R_y\hat{j}+R_z\hat{k}$$

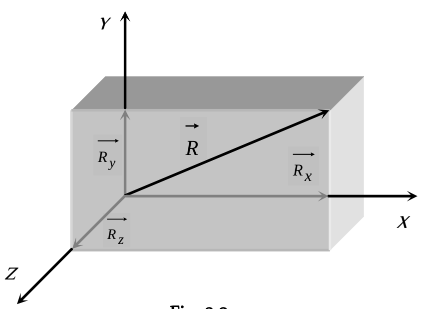


Fig. 0.9

If $\vec{R}$ makes an angle α with x axis, β with y axis and γ with z axis, then

$$\Rightarrow \cos\alpha=\frac{R_x}{R}=\frac{R_x}{\sqrt{R_x^2+R_y^2+R_z^2}}=l$$

$$\Rightarrow \cos\beta=\frac{R_y}{R}=\frac{R_y}{\sqrt{R_x^2+R_y^2+R_z^2}}=m$$

$$\Rightarrow \cos\gamma=\frac{R_z}{R}=\frac{R_z}{\sqrt{R_x^2+R_y^2+R_z^2}}=n$$

Where *l, m, n* are called Direction Cosines of the vector $\vec{R}$ and

$$l^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{R_x^2+R_y^2+R_z^2}{R_x^2+R_y^2+R_z^2}=1$$

Note : ❑ When a point P have coordinate (x, y, z) then its position vector $\overrightarrow{OP}=x\hat{i}+y\hat{j}+z\hat{k}$

❑ When a particle moves from point (x, y, z) to (x, y, z) then its displacement vector

$$\vec{r}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$

Scalar Product of Two Vectors

(1) **Definition :** The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}.\vec{B}$ is defined as $\vec{A}.\vec{B}$ $=AB\cos\theta$

(2) **Properties :** (i) It is always a scalar which is positive if angle between the vectors is acute (*i.e.*, < 90°) and negative if angle between them is obtuse (*i.e.* 90°<θ < 180°).

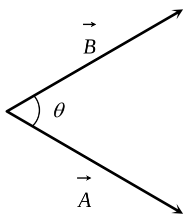


Fig. 0.10

(ii) It is commutative, *i.e.* $\vec{A}.\vec{B}=\vec{B}.\vec{A}$

(iii) It is distributive, *i.e.* $\vec{A}.(\vec{B}+\vec{C})=\vec{A}.\vec{B}+\vec{A}.\vec{C}$

(iv) As by definition $\vec{A}.\vec{B}=AB\cos\theta$

The angle between the vectors $\theta=\cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$

(v) Scalar product of two vectors will be maximum when $\cos\theta=\max=1$, *i.e.* $\theta=0^o$, *i.e.*, vectors are parallel

$$(\vec{A}.\vec{B})_{\max}=AB$$

(vi) Scalar product of two vectors will be minimum when $|\cos\theta|=\min=0$, *i.e.* $\theta=90^o$

$$(\vec{A}.\vec{B})_{\min}=0$$

i.e. if the scalar product of two nonzero vectors vanishes the vectors are orthogonal.

(vii) The scalar product of a vector by itself is termed as self dot product and is given by $(\vec{A})^2=\vec{A}.\vec{A}=AA\cos\theta=A^2$

i.e. $A=\sqrt{\vec{A}.\vec{A}}$

(viii) In case of unit vector $\hat{n}$

$\hat{n}.\hat{n}=1\times1\times\cos 0=1$ so $\hat{n}.\hat{n}=\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1$

(ix) In case of orthogonal unit vectors $\hat{i},\hat{j}$ and $\hat{k}$, $\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=1\times1\cos 90^\circ=0$

(x) In terms of components

$$\vec{A}.\vec{B}=(\hat{i}A_x+\hat{j}A_y+\hat{k}A_z).(\hat{i}B_x+\hat{j}B_y+\hat{k}B_z)=[A_xB_x+A_yB_y+A_zB_z]$$

(3) **Example :** (i) Work W : In physics for constant force work is defined as, $W=Fs\cos\theta$ ...(i)

But by definition of scalar product of two vectors, $\vec{F}.\vec{s}=Fs\cos\theta$ ...(ii)

So from *eq* (i) and (ii) $W=\vec{F}.\vec{s}$ *i.e.* work is the scalar product of force with displacement.

(ii) Power P :

As $W=\vec{F}.\vec{s}$ or $\frac{dW}{dt}=\vec{F}.\frac{d\vec{s}}{dt}$ [As $\vec{F}$ is constant]

or $P=\vec{F}.\vec{v}$ *i.e.*, power is the scalar product of force with velocity. $\left[\text{As}\frac{dW}{dt}=P \text{ and } \frac{d\vec{s}}{dt}=\vec{v}\right]$

(iii) Magnetic Flux ϕ :

Magnetic flux through an area is given by $d\phi=B\,ds\cos\theta$ ...(i)

But by definition of scalar product $\vec{B}.d\vec{s}=Bds\cos\theta$ ...(ii)

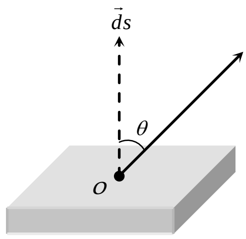


Fig. 0.11

So from *eq* (i) and (ii) we have

$$d\phi=\vec{B}.d\vec{s} \text{ or } \phi=\int\vec{B}.d\vec{s}$$

(iv) Potential energy of a dipole U : If an electric dipole of moment $\vec{p}$ is situated in an electric field $\vec{E}$ or a magnetic dipole of moment $\vec{M}$ in a field of induction $\vec{B}$, the potential energy of the dipole is given by :

$$U_E=-\vec{p}.\vec{E} \text{ and } U_B=-\vec{M}.\vec{B}$$

Vector Product of Two Vectors

(1) **Definition** : The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$$\vec{C}=\vec{A}\times\vec{B}$$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A}\times\vec{B}$ is a vector $\vec{C}$ defined by

$$\vec{C}=\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$$

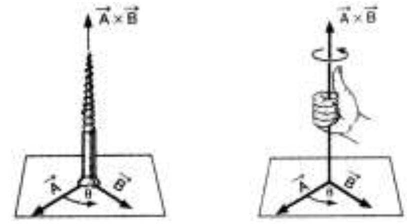


Fig. 0.12

The direction of $\vec{A}\times\vec{B}$, *i.e.* $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ *i.e.* $\vec{C}$

(2) **Properties**

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, *i.e.*, orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not commutative, *i.e.*, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ [but $=-\vec{B}\times\vec{A}$]

Here it is worthy to note that

$$|\vec{A}\times\vec{B}|=|\vec{B}\times\vec{A}|=AB\sin\theta$$

i.e. in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is distributive when the order of the vectors is strictly maintained, *i.e.*

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

(iv) The vector product of two vectors will be maximum when $\sin\theta=\text{max}=1$, *i.e.*, $\theta=90^\circ$

$$[\vec{A}\times\vec{B}]_{\text{max}}=AB\,\hat{n}$$

i.e. vector product is maximum if the vectors are orthogonal.

(v) The vector product of two non- zero vectors will be minimum when $|\sin\theta|=$ minimum $=0$, *i.e.*, $\theta=0^\circ$ or 180°

$$[\vec{A}\times\vec{B}]_{\text{min}}=0$$

i.e. if the vector product of two non-zero vectors vanishes, the vectors are collinear.

(vi) The self cross product, *i.e.*, product of a vector by itself vanishes, *i.e.*, is null vector $\vec{A}\times\vec{A}=AA\sin 0^\circ\,\hat{n}=\vec{0}$

(vii) In case of unit vector $\hat{n}\times\hat{n}=\vec{0}$ so that $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$

(viii) In case of orthogonal unit vectors, $\hat{i},\hat{j},\hat{k}$ in accordance with right hand screw rule :

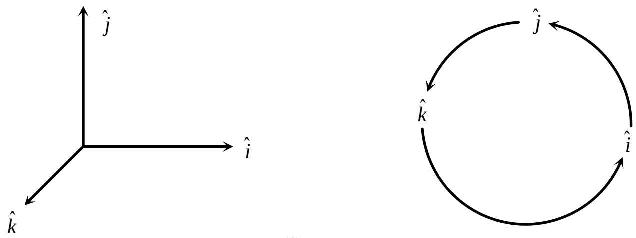


Fig. 0.13

$\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{k}=\hat{i}$ and $\hat{k}\times\hat{i}=\hat{j}$

And as cross product is not commutative,

$\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$ and $\hat{i}\times\hat{k}=-\hat{j}$

(x) In terms of components

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=\hat{i}(A_yB_z-A_zB_y)+\hat{j}(A_zB_x-A_xB_z)+\hat{k}(A_xB_y-A_yB_x)$$

(3) **Example :** Since vector product of two vectors is a vector, vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field and can be expressed as the vector product of two vectors. It is well – established in physics that :

(i) Torque $\vec{\tau}=\vec{r}\times\vec{F}$

(ii) Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

(iii) Velocity $\vec{v}=\vec{\omega}\times\vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F}=q(\vec{v}\times\vec{B})$

(v) Torque on a dipole in a field $\vec{\tau_E}=\vec{p}\times\vec{E}$ and $\vec{\tau_B}=\vec{M}\times\vec{B}$

Lami's Theorem

In any ΔABC with sides $\vec{a},\vec{b},\vec{c}$

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

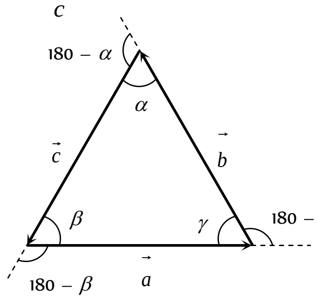


Fig. 0.14

i.e. for any triangle the ratio of the sine of the angle containing the side to the length of the side is a constant.

For a triangle whose three sides are in the same order we establish the Lami's theorem in the following manner. For the triangle shown

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$ [All three sides are taken in order] ...(i)

$\Rightarrow\ \vec{a}+\vec{b}=-\vec{c}$ ...(ii)

Pre-multiplying both sides by $\vec{a}$

$\vec{a}\times(\vec{a}+\vec{b})=-\vec{a}\times\vec{c}\ \Rightarrow\ \vec{0}+\vec{a}\times\vec{b}=-\vec{a}\times\vec{c}$

$\Rightarrow\ \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ ...(iii)

Pre-multiplying both sides of (ii) by $\vec{b}$

$\vec{b}\times(\vec{a}+\vec{b})=-\vec{b}\times\vec{c}\ \Rightarrow\ \vec{b}\times\vec{a}+\vec{b}\times\vec{b}=-\vec{b}\times\vec{c}$

$\Rightarrow\ -\vec{a}\times\vec{b}=-\vec{b}\times\vec{c}\Rightarrow\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$ ...(iv)

From (iii) and (iv), we get $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Taking magnitude, we get $|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$

$\Rightarrow\ ab\sin(180-\gamma)=bc\sin(180-\alpha)=ca\sin(180-\beta)$

$\Rightarrow\ ab\sin\gamma=bc\sin\alpha=ca\sin\beta$

Dividing through out by *abc,* we have

$$\Rightarrow\ \frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Relative Velocity

(1) **Introduction :** When we consider the motion of a particle, we assume a fixed point relative to which the given particle is in motion. For example, if we say that water is flowing or wind is blowing or a person is running with a speed *v*, we mean that these all are relative to the earth (which we have assumed to be fixed).

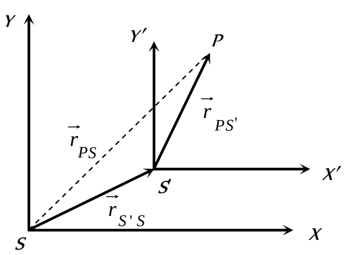


Fig. 0.15

Now to find the velocity of a moving object relative to another moving object, consider a particle *P* whose position relative to frame *S* is $\vec{r}_{PS}$ while relative to S' is $\vec{r}_{PS'}$.

If the position of frames S' relative to *S* at any time is $\vec{r}_{S'S}$ then from figure, $\vec{r}_{PS}=\vec{r}_{PS'}+\vec{r}_{S'S}$

Differentiating this equation with respect to time

$$\frac{d\vec{r}_{PS}}{dt}=\frac{d\vec{r}_{PS'}}{dt}+\frac{d\vec{r}_{S'S}}{dt}$$

or $\vec{v}_{PS}=\vec{v}_{PS'}+\vec{v}_{S'S}$ [as $\vec{v}=d\vec{r}/dt$]

or $\vec{v}_{PS'}=\vec{v}_{PS}-\vec{v}_{S'S}$

(2) **General Formula :** The relative velocity of a particle P_1 moving with velocity $\vec{v}_1$ with respect to another particle P_2 moving with velocity $\vec{v}_2$ is given by, $\vec{v}_{r_{12}} = \vec{v}_1 - \vec{v}_2$

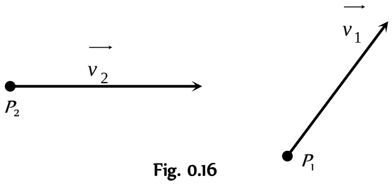


Fig. 0.16

(i) If both the particles are moving in the same direction then :

$\upsilon_{r_{12}} = \upsilon_1 - \upsilon_2$

(ii) If the two particles are moving in the opposite direction, then :

$\upsilon_{r_{12}} = \upsilon_1 + \upsilon_2$

(iii) If the two particles are moving in the mutually perpendicular directions, then:

$$\upsilon_{r_{12}} = \sqrt{\upsilon_1^2 + \upsilon_2^2}$$

(iv) If the angle between $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ be θ, then $\upsilon_{r_{12}} = \left[\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2\cos\theta\right]^{1/2}$.

(3) **Relative velocity of satellite :** If a satellite is moving in equatorial plane with velocity $\vec{v}_s$ and a point on the surface of earth with $\vec{v}_e$ relative to the centre of earth, the velocity of satellite relative to the surface of earth

$$\vec{v}_{se} = \vec{v}_s - \vec{v}_e$$

So if the satellite moves form west to east (in the direction of rotation of earth on its axis) its velocity relative to earth's surface will be $v_{se} = v_s - v_e$

And if the satellite moves from east to west, *i.e.,* opposite to the motion of earth, $v_{se} = v_s - (-v_e) = v_s + v_e$

(4) **Relative velocity of rain :** If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

which by law of vector addition has magnitude

$$v_{RM} = \sqrt{v_R^2 + v_M^2}$$

direction $\theta = \tan^{-1}(v_M / v_R)$ with the vertical as shown in fig.

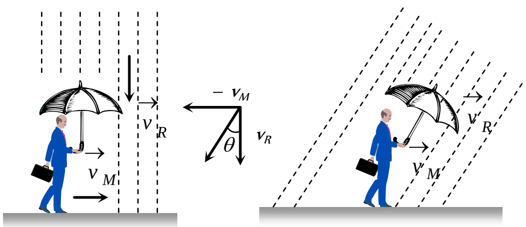


Fig. 0.17

(5) **Relative velocity of swimmer :** If a man can swim relative to water with velocity $\vec{v}$ and water is flowing relative to ground with velocity $\vec{v}_R$ velocity of man relative to ground $\vec{v}_M$ will be given by:

$$\vec{v} = \vec{v}_M - \vec{v}_R \text{ , i.e., } \vec{v}_M = \vec{v} + \vec{v}_R$$

So if the swimming is in the direction of flow of water, $v_M = v + v_R$

And if the swimming is opposite to the flow of water, $v_M = v - v_R$

(6) **Crossing the river :** Suppose, the river is flowing with velocity $\vec{\upsilon}_r$. A man can swim in still water with velocity $\vec{\upsilon}_m$. He is standing on one bank of the river and wants to cross the river, two cases arise.

(i) To cross the river over shortest distance : That is to cross the river straight, the man should swim making angle θ with the upstream as shown.

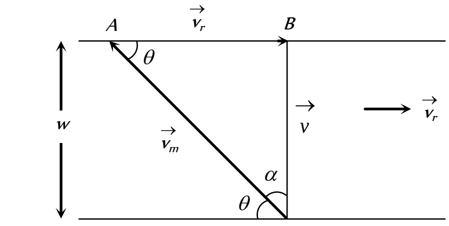


Fig. 0.18

Here OAB is the triangle of vectors, in which $\vec{OA} = \vec{v}_m, \vec{AB} = \vec{\upsilon}_r$.

Their resultant is given by $\vec{OB} = \vec{\upsilon}$. The direction of swimming makes angle θ with upstream. From the triangle OBA, we find,

$$\cos\theta = \frac{\upsilon_r}{\upsilon_m} \quad \text{Also} \quad \sin\alpha = \frac{\upsilon_r}{\upsilon_m}$$

Where α is the angle made by the direction of swimming with the shortest distance (OB) across the river.

Time taken to cross the river : If w be the width of the river, then time taken to cross the river will be given by

$$t_1 = \frac{w}{\upsilon} = \frac{w}{\sqrt{\upsilon_m^2 - \upsilon_r^2}}$$

(ii) To cross the river in shortest possible time : The man should swim perpendicular to the bank.

The time taken to cross the river will be:

$$t_2 = \frac{w}{\upsilon_m}$$

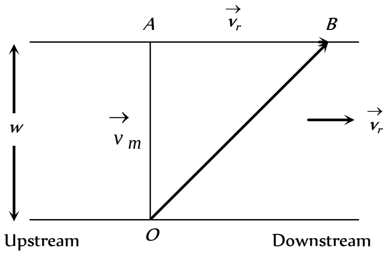


Fig. 0.19

In this case, the man will touch the opposite bank at a distance AB down stream. This distance will be given by:

$$AB = \upsilon_r t_2 = \upsilon_r \frac{w}{\upsilon_m} \quad \text{or} \quad AB = \frac{\upsilon_r}{\upsilon_m} w$$

Tips & Tricks

- All physical quantities having direction are not vectors. For example, the electric current possesses direction but it is a scalar quantity because it can not be added or multiplied according to the rules of vector algebra.
- A vector can have only two rectangular components in plane and only three rectangular components in space.
- A vector can have any number, even infinite components. (minimum 2 components)
- Following quantities are neither vectors nor scalars : Relative density, density, viscosity, frequency, pressure, stress, strain, modulus of elasticity, poisson's ratio, moment of inertia, specific heat, latent heat, spring constant loudness, resistance, conductance, reactance, impedance, permittivity, dielectric constant, permeability, susceptibility, refractive index, focal length, power of lens, Boltzman constant, Stefan's constant, Gas constant, Gravitational constant, Rydberg constant, Planck's constant etc.
- Distance covered is a scalar quantity.
- The displacement is a vector quantity.
- Scalars are added, subtracted or divided algebraically.
- Vectors are added and subtracted geometrically.
- Division of vectors is not allowed as directions cannot be divided.
- Unit vector gives the direction of vector.
- Magnitude of unit vector is 1.
- Unit vector has no unit. For example, velocity of an object is 5 ms^{-1} due East.

i.e. $\vec{v} = 5ms^{-1}$ due east.

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{5ms^{-1}(\text{East})}{5ms^{-1}} = \text{East}$$

So unit vector $\hat{v}$ has no unit as East is not a physical quantity.

- Unit vector has no dimensions.
- $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$
- $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$
- $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
- $\vec{A} \times \vec{A} = \vec{0}$. Also $\vec{A} - \vec{A} = \vec{0}$ But $\vec{A} \times \vec{A} \neq \vec{A} - \vec{A}$

Because $\vec{A} \times \vec{A} \perp \vec{A}$ and $\vec{A} - \vec{A}$ is collinear with $\vec{A}$

- Multiplication of a vector with –1 reverses its direction.

If $\vec{A} = \vec{B}$, then $A = B$ and $\hat{A} = \hat{B}$.

If $\vec{A} + \vec{B} = \vec{0}$, then $A = B$ but $\hat{A} = -\hat{B}$.

- Minimum number of collinear vectors whose resultant can be zero is two.
- Minimum number of coplaner vectors whose resultant is zero is three.
- Minimum number of non coplaner vectors whose resultant is zero is four.
- Two vectors are perpendicular to each other if $\vec{A}.\vec{B} = 0$.
- Two vectors are parallel to each other if $\vec{A} \times \vec{B} = 0$.
- Displacement, velocity, linear momentum and force are polar vectors.
- Angular velocity, angular acceleration, torque and angular momentum are axial vectors.
- Division with a vector is not defined because it is not possible to divide with a direction.
- Distance covered is always positive quantity.
- The components of a vectors can have magnitude than that of the vector itself.
- The rectangular components cannot have magnitude greater than that of the vector itself.
- When we multiply a vector with 0 the product becomes a null vector.
- The resultant of two vectors of unequal magnitude can never be a null vector.
- Three vectors not lying in a plane can never add up to give a null vector.
- A quantity having magnitude and direction is not necessarily a vector. For example, time and electric current. These quantities have magnitude and direction but they are scalar. This is because they do not obey the laws of vector addition.
- A physical quantity which has different values in different directions is called a tensor. For example : Moment of inertia has different values in different directions. Hence moment of inertia is a tensor. Other examples of tensor are refractive index, stress, strain, density etc.
- The magnitude of rectangular components of a vector is always less than the magnitude of the vector
- If $\vec{A} = \vec{B}$, then $A_x = B_x$, $A_y = B_y$ and $A_z = B_z$.
- If $\vec{A} + \vec{B} = \vec{C}$. Or if $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then $\vec{A}$, $\vec{B}$ and $\vec{C}$ lie in one plane.
- If $\vec{A} \times \vec{B} = \vec{C}$, then $\vec{C}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$.
- If $|\vec{A} \times \vec{B}| = |\vec{A} - \vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is 90°.
- Resultant of two vectors will be maximum when $\theta = 0°$ *i.e.* vectors

are parallel.

$R_{max} = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ} = |P + Q|$

✍ Resultant of two vectors will be minimum when θ = 180° *i.e.* vectors are anti-parallel.

$R_{min} = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ} = |P - Q|$

Thus, minimum value of the resultant of two vectors is equal to the difference of their magnitude.

✍ Thus, maximum value of the resultant of two vectors is equal to the sum of their magnitude.

When the magnitudes of two vectors are unequal, then

$R_{min} = P - Q \neq 0$

$[\because |\vec{P}| \neq |\vec{Q}|]$

Thus, two vectors $\vec{P}$ and $\vec{Q}$ having different magnitudes can never be combined to give zero resultant. From here, we conclude that the minimum number of vectors of unequal magnitude whose resultant can be zero is three. On the other hand, the minimum number of vectors of equal magnitude whose resultant can be zero is two.

✍ Angle between two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$$

✍ Projection of a vector $\vec{A}$ in the direction of vector $\vec{B}$

$$= \frac{\vec{A}.\vec{B}}{|\vec{B}|}$$

✍ Projection of a vector $\vec{B}$ in the direction of vector $\vec{A}$

$$= \frac{\vec{A}.\vec{B}}{|\vec{A}|}$$

✍ If vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are represented by three sides *ab, bc* and *ca* respectively taken in a order, then

$$\frac{|\vec{A}|}{ab} = \frac{|\vec{B}|}{bc} = \frac{|\vec{C}|}{ca}$$

✍ The vectors $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the coordinate axes at an angle of 54.74 degrees.

✍ If $\vec{A} \pm \vec{B} = \vec{C}$, then $\vec{A}.\vec{B} \times \vec{C} = 0$.

✍ If $\vec{A}.\vec{B} \times \vec{C} = 0$, then $\vec{A}.\vec{B}$ and $\vec{C}$ are coplanar.

✍ If angle between $\vec{A}$ and $\vec{B}$ is 45°,

then $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$

✍ If $\vec{A}_1 + \vec{A}_2 + \vec{A}_3 + ... + \vec{A}_n = \vec{0}$ and $A_1 = A_2 = A_3 = ... = A_n$ then the adjacent vector are inclined to each other at angle $2\pi / n$.

✍ If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$, then the angle between $\vec{A}$ and $\vec{B}$ is 90°. Also *A, B* and *C* can have the following values.

(i) $A = 3$, $B = 4$, $C = 5$

(ii) $A = 5$, $B = 12$, $C = 13$

(iii) $A = 8$, $B = 15$, $C = 17$.

Ordinary Thinking

Objective Questions

Fundamentals of Vectors

1. The vector projection of a vector $3\hat{i}+4\hat{k}$ on y-axis is [RPMT 2004]
   (a) 5 (b) 4
   (c) 3 (d) Zero

2. Position of a particle in a rectangular-co-ordinate system is (3, 2, 5). Then its position vector will be
   (a) $3\hat{i}+5\hat{j}+2\hat{k}$ (b) $3\hat{i}+2\hat{j}+5\hat{k}$
   (c) $5\hat{i}+3\hat{j}+2\hat{k}$ (d) None of these

3. If a particle moves from point P (2,3,5) to point Q (3,4,5). Its displacement vector be
   (a) $\hat{i}+\hat{j}+10\hat{k}$ (b) $\hat{i}+\hat{j}+5\hat{k}$
   (c) $\hat{i}+\hat{j}$ (d) $2\hat{i}+4\hat{j}+6\hat{k}$

4. A force of 5 N acts on a particle along a direction making an angle of 60° with vertical. Its vertical component be
   (a) 10 N (b) 3 N
   (c) 4 N (d) 2.5 N

5. If $A=3\hat{i}+4\hat{j}$ and $B=7\hat{i}+24\hat{j}$, the vector having the same magnitude as B and parallel to A is
   (a) $5\hat{i}+20\hat{j}$ (b) $15\hat{i}+10\hat{j}$
   (c) $20\hat{i}+15\hat{j}$ (d) $15\hat{i}+20\hat{j}$

6. Vector $\vec{A}$ makes equal angles with x, y and z axis. Value of its components (in terms of magnitude of $\vec{A}$) will be
   (a) $\frac{A}{\sqrt{3}}$ (b) $\frac{A}{\sqrt{2}}$
   (c) $\sqrt{3}A$ (d) $\frac{\sqrt{3}}{A}$

7. If $\vec{A}=2\hat{i}+4\hat{j}-5\hat{k}$ the direction of cosines of the vector $\vec{A}$ are
   (a) $\frac{2}{\sqrt{45}},\frac{4}{\sqrt{45}}$ and $\frac{-5}{\sqrt{45}}$ (b) $\frac{1}{\sqrt{45}},\frac{2}{\sqrt{45}}$ and $\frac{3}{\sqrt{45}}$
   (c) $\frac{4}{\sqrt{45}}$, 0 and $\frac{4}{\sqrt{45}}$ (d) $\frac{3}{\sqrt{45}},\frac{2}{\sqrt{45}}$ and $\frac{5}{\sqrt{45}}$

8. The vector that must be added to the vector $\hat{i}-3\hat{j}+2\hat{k}$ and $3\hat{i}+6\hat{j}-7\hat{k}$ so that the resultant vector is a unit vector along the y-axis is
   (a) $4\hat{i}+2\hat{j}+5\hat{k}$ (b) $-4\hat{i}-2\hat{j}+5\hat{k}$
   (c) $3\hat{i}+4\hat{j}+5\hat{k}$ (d) Null vector

9. How many minimum number of coplanar vectors having different magnitudes can be added to give zero resultant
   (a) 2 (b) 3
   (c) 4 (d) 5

10. A hall has the dimensions $10\,m \times 12\,m \times 14\,m$. A fly starting at one corner ends up at a diametrically opposite corner. What is the magnitude of its displacement
    (a) 17 m (b) 26 m
    (c) 36 m (d) 20 m

11. 100 coplanar forces each equal to 10 N act on a body. Each force makes angle $\pi/50$ with the preceding force. What is the resultant of the forces
    (a) 1000 N (b) 500 N
    (c) 250 N (d) Zero

12. The magnitude of a given vector with end points (4, – 4, 0) and (– 2, – 2, 0) must be
    (a) 6 (b) $5\sqrt{2}$
    (c) 4 (d) $2\sqrt{10}$

13. The expression $\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)$ is a
    (a) Unit vector (b) Null vector
    (c) Vector of magnitude $\sqrt{2}$ (d) Scalar

14. Given vector $\vec{A}=2\hat{i}+3\hat{j}$, the angle between $\vec{A}$ and y-axis is [CPMT 1993]
    (a) $\tan^{-1}3/2$ (b) $\tan^{-1}2/3$
    (c) $\sin^{-1}2/3$ (d) $\cos^{-1}2/3$

15. The unit vector along $\hat{i}+\hat{j}$ is
    (a) $\hat{k}$ (b) $\hat{i}+\hat{j}$
    (c) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{2}$

16. A vector is represented by $3\hat{i}+\hat{j}+2\hat{k}$. Its length in XY plane is
    (a) 2 (b) $\sqrt{14}$
    (c) $\sqrt{10}$ (d) $\sqrt{5}$

17. Five equal forces of 10 N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be [CBSE PMT 1995]
    (a) Zero (b) 10 N
    (c) 20 N (d) $10\sqrt{2}N$

18. The angle made by the vector $A=\hat{i}+\hat{j}$ with x- axis is [EAMCET (Engg.) 1999]
    (a) 90° (b) 45°
    (c) 22.5° (d) 30°

19. Any vector in an arbitrary direction can always be replaced by two (or three)
    (a) Parallel vectors which have the original vector as their resultant
    (b) Mutually perpendicular vectors which have the original vector as their resultant

(c) Arbitrary vectors which have the original vector as their resultant

(d) It is not possible to resolve a vector

20. Angular momentum is [MNR 1986]

(a) A scalar (b) A polar vector

(c) An axial vector (d) None of these

21. Which of the following is a vector

(a) Pressure (b) Surface tension

(c) Moment of inertia (d) None of these

22. If $\vec{P} = \vec{Q}$ then which of the following is NOT correct

(a) $\hat{P} = \hat{Q}$ (b) $|\vec{P}| = |\vec{Q}|$

(c) $P\hat{Q} = Q\hat{P}$ (d) $\vec{P} + \vec{Q} = \hat{P} + \hat{Q}$

23. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is [CBSE PMT 1995]

(a) Parallel to the position vector

(b) Perpendicular to the position vector

(c) Directed towards the origin

(d) Directed away from the origin

24. Which of the following is a scalar quantity [AFMC 1998]

(a) Displacement (b) Electric field

(c) Acceleration (d) Work

25. If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$, then the value of '$c$' is [CBSE PMT 1999; EAMCET 1994]

(a) 1 (b) $\sqrt{0.11}$

(c) $\sqrt{0.01}$ (d) $\sqrt{0.39}$

26. A boy walks uniformally along the sides of a rectangular park of size 400 $m\times$ 300 m, starting from one corner to the other corner diagonally opposite. Which of the following statement is incorrect [HP PMT 1999]

(a) He has travelled a distance of 700 m

(b) His displacement is 700 m

(c) His displacement is 500 m

(d) His velocity is not uniform throughout the walk

27. The unit vector parallel to the resultant of the vectors $\vec{A} = 4\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} - 8\hat{k}$ is [EAMCET 2000]

(a) $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (b) $\frac{1}{7}(3\hat{i} + 6\hat{j} + 2\hat{k})$

(c) $\frac{1}{49}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (d) $\frac{1}{49}(3\hat{i} - 6\hat{j} + 2\hat{k})$

28. Surface area is [J&K CET 2002]

(a) Scalar (b) Vector

(c) Neither scalar nor vector (d) Both scalar and vector

29. With respect to a rectangular cartesian coordinate system, three vectors are expressed as

$\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the X, Y and Z-axis respectively. The unit vectors $\hat{r}$ along the direction of sum of these vector is [Kerala CET (Engg.) 2003]

(a) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (b) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} - \hat{k})$

(c) $\hat{r} = \frac{1}{3}(\hat{i} - \hat{j} + \hat{k})$ (d) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

30. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ is [DPMT 2000]

(a) 60° (b) Zero

(c) 90° (d) None of these

31. The position vector of a particle is determined by the expression $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$

The distance traversed in first 10 sec is [DPMT 2002]

(a) 500 m (b) 300 m

(c) 150 m (d) 100 m

32. Unit vector parallel to the resultant of vectors $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 8\hat{i} + 8\hat{j}$ will be [BHU 1995]

(a) $\frac{24\hat{i} + 5\hat{j}}{13}$ (b) $\frac{12\hat{i} + 5\hat{j}}{13}$

(c) $\frac{6\hat{i} + 5\hat{j}}{13}$ (d) None of these

33. The component of vector $A = 2\hat{i} + 3\hat{j}$ along the vector $\hat{i} + \hat{j}$ is [KCET 1997]

(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$

(c) $5\sqrt{2}$ (d) 5

34. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be [Pb. CET 2001]

(a) 90° (b) 0°

(c) 60° (d) 45°

Addition and Subtraction of Vectors

1. There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively

(a) 0°, 180° and 90° (b) 0°, 90° and 180°

(c) 0°, 90° and 90° (d) 180°, 0° and 90°

2. If $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$ then magnitude and direction of $\vec{A} + \vec{B}$ will be

(a) $5, \tan^{-1}(3/4)$ (b) $5\sqrt{5}, \tan^{-1}(1/2)$

(c) $10, \tan^{-1}(5)$ (d) $25, \tan^{-1}(3/4)$

3. A truck travelling due north at 20 *m/s* turns west and travels at the same speed. The change in its velocity be [UPSEAT 1999]

(a) 40 *m/s N–W* (b) $20\sqrt{2}$ *m/s N–W*

(c) 40 *m/s S–W* (d) $20\sqrt{2}$ *m/s S–W*

4. If the sum of two unit vectors is a unit vector, then magnitude of difference is [CPMT 1995; CBSE PMT 1989]

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) $1/\sqrt{2}$ (d) $\sqrt{5}$

5. $\vec{A} = 2\hat{i} + \hat{j}, B = 3\hat{j} - \hat{k}$ and $\vec{C} = 6\hat{i} - 2\hat{k}$.

Value of $\vec{A} - 2\vec{B} + 3\vec{C}$ would be

(a) $20\hat{i} + 5\hat{j} + 4\hat{k}$ (b) $20\hat{i} - 5\hat{j} - 4\hat{k}$

(c) $4\hat{i} + 5\hat{j} + 20\hat{k}$ (d) $5\hat{i} + 4\hat{j} + 10\hat{k}$

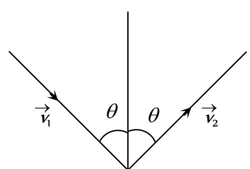
6. An object of m *kg* with speed of v *m/s* strikes a wall at an angle θ and rebounds at the same speed and same angle. The magnitude of the change in momentum of the object will be

(a) $2mv\cos\theta$

(b) $2mv\sin\theta$

(c) 0

(d) $2mv$



7. Two forces, each of magnitude [illegible]ne magnitude F. The angle between the two forces is [CBSE PMT 1990]

(a) 45° (b) 120°

(c) 150° (d) 60°

8. For the resultant of the two vectors to be maximum, what must be the angle between them

(a) 0° (b) 60°

(c) 90° (d) 180°

9. A particle is simultaneously acted by two forces equal to 4 *N* and 3 *N*. The net force on the particle is [CPMT 1979]

(a) 7 *N* (b) 5 *N*

(c) 1 *N* (d) Between 1 *N* and 7 *N*

10. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane, another vector $\vec{C}$ lies outside this plane, then the resultant of these three vectors *i.e.*, $\vec{A} + \vec{B} + \vec{C}$

(a) Can be zero

(b) Cannot be zero

(c) Lies in the plane containing $\vec{A} + \vec{B}$

(d) Lies in the plane containing $\vec{C}$

11. If the resultant of the two forces has a magnitude smaller than the magnitude of larger force, the two forces must be

(a) Different both in magnitude and direction

(b) Mutually perpendicular to one another

(c) Possess extremely small magnitude

(d) Point in opposite directions

12. Forces F_1 and F_2 act on a point mass in two mutually perpendicular directions. The resultant force on the point mass will be [CPMT 1991]

(a) $F_1 + F_2$ (b) $F_1 - F_2$

(c) $\sqrt{F_1^2 + F_2^2}$ (d) $F_1^2 + F_2^2$

13. If $|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}|$, the angle between $\vec{A}$ and $\vec{B}$ is

(a) 60° (b) 0°

(c) 120° (d) 90°

14. Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and resultant be $\vec{C}$

(a) $\vec{C}$ must be equal to $|\vec{A} - \vec{B}|$

(b) $\vec{C}$ must be less than $|\vec{A} - \vec{B}|$

(c) $\vec{C}$ must be greater than $|\vec{A} - \vec{B}|$

(d) $\vec{C}$ may be equal to $|\vec{A} - \vec{B}|$

15. The magnitude of vector $\vec{A}, \vec{B}$ and $\vec{C}$ are respectively 12, 5 and 13 units and $\vec{A} + \vec{B} = \vec{C}$ then the angle between $\vec{A}$ and $\vec{B}$ is

(a) 0 (b) π

(c) $\pi/2$ (d) $\pi/4$

16. Magnitude of vector which comes on addition of two vectors, $6\hat{i} + 7\hat{j}$ and $3\hat{i} + 4\hat{j}$ is [BHU 2000]

(a) $\sqrt{136}$ (b) $\sqrt{13.2}$

(c) $\sqrt{202}$ (d) $\sqrt{160}$

17. A particle has displacement of 12 *m* towards east and 5 *m* towards north then 6 *m* vertically upward. The sum of these displacements is

(a) 12 (b) 10.04 *m*

(c) 14.31 *m* (d) None of these

18. The three vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form

(a) An equilateral triangle (b) Isosceles triangle

(c) A right angled triangle (d) No triangle

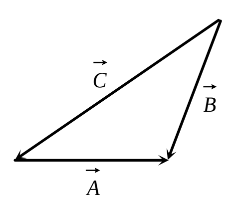
19. For the figure

(a) $\vec{A} + \vec{B} = \vec{C}$

(b) $\vec{B} + \vec{C} = \vec{A}$

(c) $\vec{C} + \vec{A} = \vec{B}$

(d) $\vec{A} + \vec{B} + \vec{C} = 0$ [CPMT 1983]



20. Let $\vec{C} = \vec{A} + \vec{B}$ then

(a) $|\vec{C}|$ is always greater then $|\vec{A}|$

(b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$

(c) C is always equal to $A + B$

(d) C is never equal to $A + B$

21. The value of the sum of two vectors $\vec{A}$ and $\vec{B}$ with θ as the angle between them is [BHU 1996]

(a) $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ (b) $\sqrt{A^2 - B^2 + 2AB\cos\theta}$

(c) $\sqrt{A^2 + B^2 - 2AB\sin\theta}$ (d) $\sqrt{A^2 + B^2 + 2AB\sin\theta}$

22. Following sets of three forces act on a body. Whose resultant cannot be zero [CPMT 1985]

(a) 10, 10, 10 (b) 10, 10, 20

(c) 10, 20, 23 (d) 10, 20, 40

23. When three forces of 50 *N*, 30 *N* and 15 *N* act on a body, then the body is

(a) At rest

(b) Moving with a uniform velocity

(c) In equilibrium

(d) Moving with an acceleration

24. The sum of two forces acting at a point is 16 *N*. If the resultant force is 8 *N* and its direction is perpendicular to minimum force then the forces are [CPMT 1997]

(a) 6 *N* and 10 *N* (b) 8 *N* and 8 *N*

(c) 4 *N* and 12 *N* (d) 2 *N* and 14 *N*

25. If vectors *P*, *Q* and *R* have magnitude 5, 12 and 13 units and $\vec{P} + \vec{Q} = \vec{R}$, the angle between *Q* and *R* is [CEET 1998]

(a) $\cos^{-1}\frac{5}{12}$ (b) $\cos^{-1}\frac{5}{13}$

(c) $\cos^{-1}\frac{12}{13}$ (d) $\cos^{-1}\frac{7}{13}$

26. The resultant of two vectors *A* and *B* is perpendicular to the vector *A* and its magnitude is equal to half the magnitude of vector *B*. The angle between *A* and *B* is

(a) 120° (b) 150°

(c) 135° (d) None of these

27. What vector must be added to the two vectors $\hat{i} - 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$, so that the resultant may be a unit vector along *x*-axis [BHU 1990]

(a) $2\hat{i} + \hat{j} - \hat{k}$ (b) $-2\hat{i} + \hat{j} - \hat{k}$

(c) $2\hat{i} - \hat{j} + \hat{k}$ (d) $-2\hat{i} - \hat{j} - \hat{k}$

28. What is the angle between $\vec{P}$ and the resultant of $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$

(a) Zero (b) $\tan^{-1}(P/Q)$

(c) $\tan^{-1}(Q/P)$ (d) $\tan^{-1}(P-Q)/(P+Q)$

29. The resultant of $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. What is the angle between $\vec{P}$ and $\vec{Q}$

(a) $\cos^{-1}(P/Q)$ (b) $\cos^{-1}(-P/Q)$

(c) $\sin^{-1}(P/Q)$ (d) $\sin^{-1}(-P/Q)$

30. Maximum and minimum magnitudes of the resultant of two vectors of magnitudes *P* and *Q* are in the ratio $3:1$. Which of the following relations is true

(a) $P = 2Q$ (b) $P = Q$

(c) $PQ = 1$ (d) None of these

31. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If *Q* is doubled, the new resultant is perpendicular to *P*. Then *R* equals

(a) *P* (b) (*P*+*Q*)

(c) *Q* (d) (*P*–*Q*)

32. Two forces, F_1 and F_2 are acting on a body. One force is double that of the other force and the resultant is equal to the greater force. Then the angle between the two forces is

(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(-1/2)$

(c) $\cos^{-1}(-1/4)$ (d) $\cos^{-1}(1/4)$

33. Given that $\vec{A} + \vec{B} = \vec{C}$ and that $\vec{C}$ is $\perp$ to $\vec{A}$. Further if $|\vec{A}| = |\vec{C}|$, then what is the angle between $\vec{A}$ and $\vec{B}$

(a) $\frac{\pi}{4}$ *radian* (b) $\frac{\pi}{2}$ *radian*

(c) $\frac{3\pi}{4}$ *radian* (d) π *radian*

34. A body is at rest under the action of three forces, two of which are $\vec{F}_1 = 4\hat{i}, \vec{F}_2 = 6\hat{j}$, the third force is [AMU 1996]

(a) $4\hat{i} + 6\hat{j}$ (b) $4\hat{i} - 6\hat{j}$

(c) $-4\hat{i} + 6\hat{j}$ (d) $-4\hat{i} - 6\hat{j}$

35. A plane is revolving around the earth with a speed of 100 *km/hr* at a constant height from the surface of earth. The change in the velocity as it travels half circle is [RPET 1998; KCET 2000]

(a) 200 *km/hr* (b) 150 *km/hr*

(c) $100\sqrt{2}\ km/hr$ (d) 0

36. What displacement must be added to the displacement $25\hat{i} - 6\hat{j}\ m$ to give a displacement of 7.0 *m* pointing in the *x*-direction

(a) $18\hat{i} - 6\hat{j}$ (b) $32\hat{i} - 13\hat{j}$

(c) $-18\hat{i} + 6\hat{j}$ (d) $-25\hat{i} + 13\hat{j}$

37. A body moves due East with velocity 20 *km/hour* and then due North with velocity 15 *km/hour*. The resultant velocity [AFMC 1995]

(a) 5 *km/hour* (b) 15 *km/hour*

(c) 20 *km/hour* (d) 25 *km/hour*

38. The magnitudes of vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively. If $\vec{A} + \vec{B} = \vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is [CBSE PMT 1990]

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}(0.6)$

(c) $\tan^{-1}\left(\frac{7}{5}\right)$ (d) $\frac{\pi}{4}$

39. While travelling from one station to another, a car travels 75 *km* North, 60 *km* North-east and 20 *km* East. The minimum distance between the two stations is [AFMC 1993]

(a) 72 *km* (b) 112 *km*

(c) 132 *km* (d) 155 *km*

40. A scooter going due east at 10 *ms* turns right through an angle of 90°. If the speed of the scooter remains unchanged in taking turn, the change is the velocity of the scooter is

[BHU 1994]

(a) 20.0 ms^{-1} south eastern direction

(b) Zero

(c) 10.0 ms^{-1} in southern direction

(d) 14.14 ms^{-1} in south-west direction

41. A person goes 10 *km* north and 20 *km* east. What will be displacement from initial point [AFMC 1994, 2003]

(a) 22.36 *km* (b) 2 *km*

(c) 5 *km* (d) 20 *km*

42. Two forces $\vec{F}_1 = 5\hat{i} + 10\hat{j} - 20\hat{k}$ and $\vec{F}_2 = 10\hat{i} - 5\hat{j} - 15\hat{k}$ act on a single point. The angle between $\vec{F}_1$ and $\vec{F}_2$ is nearly

[AMU 1995]

(a) 30° (b) 45°

(c) 60° (d) 90°

43. Which pair of the following forces will never give resultant force of 2 *N* [HP PMT 1999]

(a) 2 *N* and 2 *N* (b) 1 *N* and 1 *N*

(c) 1 *N* and 3 *N* (d) 1 *N* and 4 *N*

44. Two forces 3*N* and 2 *N* are at an angle θ such that the resultant is *R*. The first force is now increased to 6*N* and the resultant become 2*R*. The value of θ is [HP PMT 2000]

(a) 30° (b) 60°

(c) 90° (d) 120°

45. Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the forces ? Also name the triangle formed by the forces as sides

[JIPMER 2000]

(a) 60° equilateral triangle

(b) 120° equilateral triangle

(c) 120°, 30°, 30° an isosceles triangle

(d) 120° an obtuse angled triangle

46. If $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be

[CBSE PMT 2001]

(a) 90° (b) 120°

(c) 0° (d) 60°

47. The maximum and minimum magnitude of the resultant of two given vectors are 17 *units* and 7 *unit* respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is [Kerala CET (Engg.) 2000]

(a) 14 (b) 16

(c) 18 (d) 13

48. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces [CBSE PMT 2003]

(a) Are equal to each other in magnitude

(b) Are not equal to each other in magnitude

(c) Cannot be predicted

(d) Are equal to each other

49. *y* component of velocity is 20 and *x* component of velocity is 10. The direction of motion of the body with the horizontal at this instant is

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(1/2)$

(c) 45° (d) 0°

50. Two forces of 12 *N* and 8 *N* act upon a body. The resultant force on the body has maximum value of [Manipal 2003]

(a) 4 *N* (b) 0 *N*

(c) 20 *N* (d) 8 *N*

51. Two equal forces (*P* each) act at a point inclined to each other at an angle of 120°. The magnitude of their resultant is

(a) $P/2$ (b) $P/4$

(c) P (d) 2*P*

52. The vectors $5i + 8j$ and $2i + 7j$ are added. The magnitude of the sum of these vector is [BHU 2000]

(a) $\sqrt{274}$ (b) 38

(c) 238 (d) 560

53. Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A} + \vec{B} = \vec{A} - \vec{B}$. Then

[AMU (Med.) 2000]

(a) $\vec{A}.\vec{B} = 0$ (b) $\vec{A} \times \vec{B} = 0$

(c) $\vec{A} = 0$ (d) $\vec{B} = 0$

Multiplication of Vectors

1. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$. Then the value of α is [CBSE PMT 2005]

(a) –1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) 1

2. If two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other then value of λ be

(a) 0 (b) 2

(c) 3 (d) 4

3. A body, acted upon by a force of 50 *N* is displaced through a distance 10 *meter* in a direction making an angle of 60° with the force. The work done by the force be

(a) 200 *J* (b) 100 *J*

(c) 300 (d) 250 *J*

4. A particle moves from position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to a uniform force of $(4\hat{i} + \hat{j} + 3\hat{k})N$. If the displacement in meters then work done will be

[CMEET 1995; Pb. PMT 2002, 03]

(a) 100 *J* (b) 200 *J*

(c) 300 *J* (d) 250 *J*

5. If for two vector $\vec{A}$ and $\vec{B}$, sum $(\vec{A} + \vec{B})$ is perpendicular to the difference $(\vec{A} - \vec{B})$. The ratio of their magnitude is

[Manipal 2003]

(a) (b) 2

(c) 3 (d) None of these

6. The angle between the vectors $\vec{A}$ and $\vec{B}$ is θ. The value of the triple product $\vec{A}.(\vec{B}\times\vec{A})$ is [CBSE PMT 1991, 2005]

(a) A^2B (b) Zero

(c) $A^2B\sin\theta$ (d) $A^2B\cos\theta$

7. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ then the angle between *A* and *B* is [AIEEE 2004]

(a) $\pi/2$ (b) $\pi/3$

(c) π (d) $\pi/4$

8. If $\vec{A}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4\hat{k}$ then value of $|\vec{A}\times\vec{B}|$ will be

(a) $8\sqrt{2}$ (b) $8\sqrt{3}$

(c) $8\sqrt{5}$ (d) $5\sqrt{8}$

9. The torque of the force $\vec{F}=(2\hat{i}-3\hat{j}+4\hat{k})N$ acting at the point $\vec{r}=(3\hat{i}+2\hat{j}+3\hat{k})\,m$ about the origin be [CBSE PMT 1995]

(a) $6\hat{i}-6\hat{j}+12\hat{k}$ (b) $17\hat{i}-6\hat{j}-13\hat{k}$

(c) $-6\hat{i}+6\hat{j}-12\hat{k}$ (d) $-17\hat{i}+6\hat{j}+13\hat{k}$

10. If $\vec{A}\times\vec{B}=\vec{C}$, then which of the following statements is wrong

(a) $\vec{C}\perp\vec{A}$ (b) $\vec{C}\perp\vec{B}$

(c) $\vec{C}\perp(\vec{A}+\vec{B})$ (d) $\vec{C}\perp(\vec{A}\times\vec{B})$

11. If a particle of mass m is moving with constant velocity v parallel to *x*-axis in *x-y* plane as shown in fig. Its angular momentum with respect to origin at any time t will be

(a) $mvb\,\hat{k}$ (b) $-mvb\,\hat{k}$

(c) $mvb\,\hat{i}$ (d) $mv\,\hat{i}$

12. Consider two vectors $\vec{F}_1=2\hat{i}+5\hat{k}$ and $\vec{F}_2=3\hat{j}+4\hat{k}$. The magnitude of the scalar product of these vectors is [MP PMT 1987]

(a) 20 (b) 23

(c) $5\sqrt{33}$ (d) 26

13. Consider a vector $\vec{F}=4\hat{i}-3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is

(a) $4\hat{i}+3\hat{j}$ (b) $6\hat{i}$

(c) $7\hat{k}$ (d) $3\hat{i}-4\hat{j}$

14. Two vectors $\vec{A}$ and $\vec{B}$ are at right angles to each other, when

(a) $\vec{A}+\vec{B}=0$ (b) $\vec{A}-\vec{B}=0$

(c) $\vec{A}\times\vec{B}=0$ (d) $\vec{A}.\vec{B}=0$

15. If $|\vec{V}_1+\vec{V}_2|=|\vec{V}_1-\vec{V}_2|$ and V_2 is finite, then [CPMT 1989]

(a) V_1 is parallel to V_2

(b) $\vec{V}_1=\vec{V}_2$

(c) V_1 and V_2 are mutually perpendicular

(d) $|\vec{V}_1|=|\vec{V}_2|$

16. A force $\vec{F}=(5\hat{i}+3\hat{j})$ Newton is applied over a particle which displaces it from its origin to the point $\vec{r}=(2\hat{i}-1\hat{j})$ *metres*. The work done on the particle is [MP PMT 1995]

(a) $-7\,J$ (b) $+13\,J$

(c) $+7\,J$ (d) $+11\,J$

17. The angle between two vectors $-2\hat{i}+3\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$ is

(a) 0° (b) 90°

(c) 180° (d) None of the above

18. The angle between the vectors $(\hat{i}+\hat{j})$ and $(\hat{j}+\hat{k})$ is [EAMCET 1995]

(a) 30° (b) 45°

(c) 60° (d) 90°

19. A particle moves with a velocity $6\hat{i}-4\hat{j}+3\hat{k}\,m/s$ under the influence of a constant force $\vec{F}=20\hat{i}+15\hat{j}-5\hat{k}\,N$. The instantaneous power applied to the particle is [CBSE PMT 2000]

(a) 35 *J/s* (b) 45 *J/s*

(c) 25 *J/s* (d) 195 *J/s*

20. If $\vec{P}.\vec{Q}=PQ$, then angle between $\vec{P}$ and $\vec{Q}$ is [AIIMS 1999]

(a) 0° (b) 30°

(c) 45° (d) 60°

21. A force $\vec{F}=5\hat{i}+6\hat{j}+4\hat{k}$ acting on a body, produces a displacement $\vec{S}=6\hat{i}-5\hat{k}$. Work done by the force is [KCET 1999]

(a) 10 units (b) 18 units

(c) 11 units (d) 5 units

22. The angle between the two vectors $\vec{A}=5\hat{i}+5\hat{j}$ and $\vec{B}=5\hat{i}-5\hat{j}$ will be [CPMT 2000]

(a) Zero (b) 45°

(c) 90° (d) 180°

23. The vector $\vec{P}=a\hat{i}+a\hat{j}+3\hat{k}$ and $\vec{Q}=a\hat{i}-2\hat{j}-\hat{k}$ are perpendicular to each other. The positive value of a is [AFMC 2000; AIIMS 2002]

(a) 3 (b) 4

(c) 9 (d) 13

24. A body, constrained to move in the *Y*-direction is subjected to a force given by $\vec{F}=(-2\hat{i}+15\hat{j}+6\hat{k})N$. What is the work done by this force in moving the body a distance 10 m along the *Y*-axis [AIIMS 1987]

(a) 20 *J* (b) 150 *J*

(c) 160 *J* (d) 190 *J*

25. A particle moves in the x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $(\vec{P})$ at anytime t is $P_x = 2\cos t, p_y = 2\sin t$. The angle θ between $\vec{F}$ and $\vec{P}$ at a given time t. will be [MNR 1991; UPSEAT 2000]

(a) $\theta = 0°$ (b) $\theta = 30°$
(c) $\theta = 90°$ (d) $\theta = 180°$

26. The area of the parallelogram represented by the vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$ is

(a) 14 units (b) 7.5 units
(c) 10 units (d) 5 units

27. A vector $\vec{F}_1$ is along the positive X-axis. If its vector product with another vector $\vec{F}_2$ is zero then $\vec{F}_2$ could be [MP PMT 1987]

(a) $4\hat{j}$ (b) $-(\hat{i} + \hat{j})$
(c) $(\hat{j} + \hat{k})$ (d) $(-4\hat{i})$

28. If for two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} \times \vec{B} = 0$, the vectors

(a) Are perpendicular to each other
(b) Are parallel to each other
(c) Act at an angle of 60°
(d) Act at an angle of 30°

29. The angle between vectors $(\vec{A} \times \vec{B})$ and $(\vec{B} \times \vec{A})$ is

(a) Zero (b) π
(c) $\pi/4$ (d) $\pi/2$

30. What is the angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} \times \vec{Q})$

(a) 0 (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) π

31. The resultant of the two vectors having magnitude 2 and 3 is 1. What is their cross product

(a) 6 (b) 3
(c) 1 (d) 0

32. Let $\vec{A} = \hat{i}A\cos\theta + \hat{j}A\sin\theta$ be any vector. Another vector $\vec{B}$ which is normal to A is [BHU 1997]

(a) $\hat{i}B\cos\theta + jB\sin\theta$ (b) $\hat{i}B\sin\theta + jB\cos\theta$
(c) $\hat{i}B\sin\theta - jB\cos\theta$ (d) $\hat{i}B\cos\theta - jB\sin\theta$

33. The angle between two vectors given by $6\bar{i} + 6\bar{j} - 3\bar{k}$ and $7\bar{i} + 4\bar{j} + 4\bar{k}$ is [EAMCET (Engg.) 1999]

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

34. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is [UPSEAT 2000]

(a) Zero (b) Along west
(c) Along east (d) Vertically downward

35. Angle between the vectors $(\hat{i} + \hat{j})$ and $(\hat{j} - \hat{k})$ is

(a) 90° (b) 0°
(c) 180° (d) 60°

36. The position vectors of points A, B, C and D are $A = 3\hat{i} + 4\hat{j} + 5\hat{k}$, $B = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $C = 7\hat{i} + 9\hat{j} + 3\hat{k}$ and $D = 4\hat{i} + 6\hat{j}$ then the displacement vectors AB and CD are

(a) Perpendicular
(b) Parallel
(c) Antiparallel
(d) Inclined at an angle of 60°

37. If force $(\vec{F}) = 4\hat{i} + 5\hat{j}$ and displacement $(\vec{s}) = 3\hat{i} + 6\hat{k}$ then the work done is [Manipal 1995]

(a) 4×3 (b) 5×6
(c) 6×3 (d) 4×6

38. If $|\vec{A} \times \vec{B}| = |\vec{A}.\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be [AIIMS 2000; Manipal 2000]

(a) 30° (b) 45°
(c) 60° (d) 90°

39. In an clockwise system [CPMT 1990]

(a) $\hat{j} \times \hat{k} = \hat{i}$ (b) $\hat{i}.\hat{i} = 0$
(c) $\hat{j} \times \hat{j} = 1$ (d) $\hat{k}.\hat{j} = 1$

40. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body is $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and the radius vector $\vec{r} = 4\hat{j} - 3\hat{k}$, then $|\vec{v}|$ is

(a) $\sqrt{29}$ units (b) $\sqrt{31}$ units
(c) $\sqrt{37}$ units (d) $\sqrt{41}$ units

41. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the relation $\vec{a}.\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$. The vector $\vec{a}$ is parallel to [AIIMS 1996]

(a) $\vec{b}$ (b) $\vec{c}$
(c) $\vec{b}.\vec{c}$ (d) $\vec{b} \times \vec{c}$

42. The diagonals of a parallelogram are $2\hat{i}$ and $2\hat{j}$. What is the area of the parallelogram

(a) 0.5 units (b) 1 unit
(c) 2 units (d) 4 units

43. What is the unit vector perpendicular to the following vectors $2\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$

(a) $\frac{\hat{i}+10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (b) $\frac{\hat{i}-10\hat{j}+18\hat{k}}{5\sqrt{17}}$

(c) $\frac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (d) $\frac{\hat{i}+10\hat{j}+18\hat{k}}{5\sqrt{17}}$

44. The area of the parallelogram whose sides are represented by the vectors $\hat{j}+3\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$ is

(a) $\sqrt{61}$ sq.unit (b) $\sqrt{59}$ sq.unit

(c) $\sqrt{49}$ sq.unit (d) $\sqrt{52}$ sq.unit

45. The position of a particle is given by $\vec{r}=(\vec{i}+2\vec{j}-\vec{k})$ momentum $\vec{P}=(3\vec{i}+4\vec{j}-2\vec{k})$. The angular momentum is perpendicular to

(a) *x*-axis

(b) *y*-axis

(c) *z*-axis

(d) Line at equal angles to all the three axes

46. Two vector *A* and *B* have equal magnitudes. Then the vector *A* + *B* is perpendicular to

(a) $A \times B$ (b) $A - B$

(c) $3A - 3B$ (d) All of these

47. Find the torque of a force $\vec{F}=-3\hat{i}+\hat{j}+5\hat{k}$ acting at the point $\vec{r}=7\hat{i}+3\hat{j}+\hat{k}$

[CPMT 1997; CBSE PMT 1997; CET 1998; DPMT 2004]

(a) $14\hat{i}-38\hat{j}+16\hat{k}$ (b) $4\hat{i}+4\hat{j}+6\hat{k}$

(c) $21\hat{i}+4\hat{j}+4\hat{k}$ (d) $-14\hat{i}+34\hat{j}-16\hat{k}$

48. The value of $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$ is

[RPET 1991, 2002; BHU 2002]

(a) 0 (b) $A^2 - B^2$

(c) $\vec{B}\times\vec{A}$ (d) $2(\vec{B}\times\vec{A})$

49. If $\vec{A}$ and $\vec{B}$ are perpendicular vectors and vector $\vec{A}=5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{B}=2\hat{i}+2\hat{j}-a\hat{k}$. The value of *a* is

[EAMCET 1991]

(a) – 2 (b) 8

(c) – 7 (d) – 8

50. A force vector applied on a mass is represented as $\vec{F}=6\hat{i}-8\hat{j}+10\hat{k}$ and accelerates with $1\ m/s^2$. What will be the mass of the body in kg.

[CMEET 1995]

(a) $10\sqrt{2}$ (b) 20

(c) $2\sqrt{10}$ (d) 10

51. Two adjacent sides of a parallelogram are represented by the two vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$. What is the area of parallelogram

[AMU 1997]

(a) 8 (b) $8\sqrt{3}$

(c) $3\sqrt{8}$ (d) 192

52. The position vectors of radius are $2\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}+\hat{k}$ while those of linear momentum are $2\hat{i}+3\hat{j}-\hat{k}$. Then the angular momentum is [BHU 1997]

[EAMCET (Engg.) 1998]

(a) $2\hat{i}-4\hat{k}$ (b) $4\hat{i}-8\hat{k}$

(c) $2\hat{i}-4\hat{j}+2\hat{k}$ (d) $4\hat{i}-8\hat{k}$

53. What is the value of linear velocity, if $\vec{\omega}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{r}=5\hat{i}-6\hat{j}+6\hat{k}$ [CBSE PMT 1999; CPMT 1999, 2001; Pb. PMT 2000; Pb. CET 2000]

(a) $6\hat{i}-2\hat{j}+3\hat{k}$ (b) $6\hat{i}-2\hat{j}+8\hat{k}$

(c) $4\hat{i}-13\hat{j}+6\hat{k}$ (d) $-18\hat{i}-13\hat{j}+2\hat{k}$

54. Dot product of two mutual perpendicular vector is

[Haryana CEET 2002]

(a) 0 (b) 1

(c) ∞ (d) None of these

55. When $\vec{A}.\vec{B}=-|A||B|$, then [Orissa JEE 2003]

(a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other

(b) $\vec{A}$ and $\vec{B}$ act in the same direction

(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction

(d) $\vec{A}$ and $\vec{B}$ can act in any direction

56. If $|\vec{A}\times\vec{B}|=\sqrt{3}\vec{A}.\vec{B}$, then the value of $|\vec{A}+\vec{B}|$ is

[CBSE PMT 2004]

(a) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$ (b) $A+B$

(c) $(A^2+B^2+\sqrt{3}AB)^{1/2}$ (d) $(A^2+B^2+AB)^{1/2}$

57. A force $\vec{F}=3\hat{i}+c\hat{j}+2\hat{k}$ acting on a particle causes a displacement $\vec{S}=-4\hat{i}+2\hat{j}-3\hat{k}$ in its own direction. If the work done is 6*J*, then the value of *c* will be [DPMT 1997]

(a) 12 (b) 6

(c) 1 (d) 0

58. A force $\vec{F}=(5\hat{i}+3\hat{j})\ N$ is applied over a particle which displaces it from its original position to the point $\vec{s}=(2\hat{i}-1\hat{j})\ m$. The work done on the particle is [BHU 2001]

(a) + 11 *J* (b) + 7 *J*

(c) + 13 *J* (d) – 7 *J*

59. If a vector $\vec{A}$ is parallel to another vector $\vec{B}$ then the resultant of the vector $\vec{A} \times \vec{B}$ will be equal to [Pb. CET 1996]

(a) A (b) $\vec{A}$

(c) Zero vector (d) Zero

Lami's Theorem

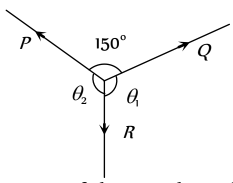
1. P, Q and R are three coplanar forces acting at a point and are in equilibrium. Given $P = 1.9318$ *kg wt*, $\sin\theta_1 = 0.9659$, the value of R is (in *kg wt*) [CET 1998]

(a) 0.9659

(b) 2

(c) 1

(d) $\frac{1}{2}$



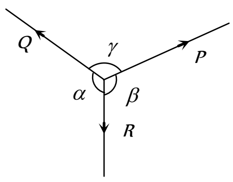
2. A body is in equilibrium under the action of three coplanar forces P, Q and R as shown in the figure. Select the correct statement

(a) $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$

(b) $\frac{P}{\cos\alpha} = \frac{Q}{\cos\beta} = \frac{R}{\cos\gamma}$

(c) $\frac{P}{\tan\alpha} = \frac{Q}{\tan\beta} = \frac{R}{\tan\gamma}$

(d) $\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$



3. If a body is in equilibrium under a set of non-collinear forces, then the minimum number of forces has to be [AIIMS 2000]

(a) Four (b) Three

(c) Two (d) Five

4. How many minimum number of non-zero vectors in different planes can be added to give zero resultant

(a) 2 (b) 3

(c) 4 (d) 5

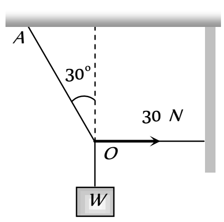
5. As shown in figure the tension in the horizontal cord is 30 N. The weight W and tension in the string OA in Newton are [DPMT 1992]

(a) $30\sqrt{3}$, 30

(b) $30\sqrt{3}$, 60

(c) $60\sqrt{3}$, 30

(d) None of these



Relative Velocity

1. Two cars are moving in the same direction with the same speed 30 *km/hr*. They are separated by a distance of 5 *km*, the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes, will be

(a) 40 *km/hr* (b) 45 *km/hr*

(c) 30 *km/hr* (d) 15 *km/hr*

2. A man standing on a road hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 *km/hr*. He finds that raindrops are hitting his head vertically, the speed of raindrops with respect to the road will be

(a) 10 *km/hr* (b) 20 *km/hr*

(c) 30 *km/hr* (d) 40 *km/hr*

3. In the above problem, the speed of raindrops *w.r.t.* the moving man, will be

(a) $10/\sqrt{2}\ km/h$ (b) 5 *km/h*

(c) $10\sqrt{3}\ km/h$ (d) $5/\sqrt{3}\ km/h$

4. A boat is moving with a velocity $3\hat{i} + 4\hat{j}$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i} - 4\hat{j}$ with respect to ground. The relative velocity of the boat with respect to water is

(a) $8\hat{j}$ (b) $-6\hat{i} - 8\hat{j}$

(c) $6\hat{i} + 8\hat{j}$ (d) $5\sqrt{2}$

5. A 150 m long train is moving to north at a speed of 10 m/s. A parrot flying towards south with a speed of 5 m/s crosses the train. The time [AFMC 1994] the parrot the cross to train would be:

(a) 30 s (b) 15 s

(c) 8 s (d) 10 s

6. A river is flowing from east to west at a speed of 5 m/min. A man on south bank of river, capable of swimming 10m/min in still water, wants to swim across the river in shortest time. He should swim

(a) Due north

(b) Due north-east

(c) Due north-east with double the speed of river

(d) None of these

7. A person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of 0.5 *m/s* at an angle of 120° with the direction of flow of water. The speed of water in the stream is [CBSE PMT 1999]

(a) 1 *m/s* (b) 0.5 *m/s*

(c) 0.25 *m/s* (d) 0.433 *m/s*

8. A moves with 65 *km/h* while B is coming back of A with 80 *km/h*. The relative velocity of B with respect to A is [AFMC 2000]

(a) 80 *km/h* (b) 60 *km/h*

(c) 15 *km/h* (d) 145 *km/h*

9. A thief is running away on a straight road on a jeep moving with a speed of 9 *m/s*. A police man chases him on a motor cycle moving at a speed of 10 *m/s*. If the instantaneous separation of jeep from the motor cycle is 100 *m*, how long will it take for the policemen to catch the thief

(a) 1 second (b) 19 second

(c) 90 second (d) 100 second

10. A man can swim with velocity v relative to water. He has to cross a river of width d flowing with a velocity u ($u > v$). The distance through which he is carried down stream by the river is x. Which of the following statement is correct

(a) If he crosses the river in minimum time $x = \frac{du}{v}$

(b) x can not be less than $\frac{du}{v}$

(c) For x to be minimum he has to swim in a direction making an angle of $\frac{\pi}{2}+\sin^{-1}\left(\frac{v}{u}\right)$ with the direction of the flow of water

(d) x will be max. if he swims in a direction making an angle of $\frac{\pi}{2}+\sin^{-1}\frac{v}{u}$ with direction of the flow of water

11. A man sitting in a bus travelling in a direction from west to east with a speed of 40 *km/h* observes that the rain-drops are falling vertically down. To the another man standing on ground the rain will appear [HP PMT 1999]

(a) To fall vertically down

(b) To fall at an angle going from west to east

(c) To fall at an angle going from east to west

(d) The information given is insufficient to decide the direction of rain.

12. A boat takes two hours to travel 8 *km* and back in still water. If the velocity of water is 4 *km/h*, the time taken for going upstream 8 *km* and coming back is [EAMCET 1990]

(a) 2*h*

(b) 2*h* 40 *min*

(c) 1*h* 20 *min*

(d) Cannot be estimated with the information given

13. A 120 *m* long train is moving towards west with a speed of 10 *m/s*. A bird flying towards east with a speed of 5 *m/s* crosses the train. The time taken by the bird to cross the train will be

(a) 16 *sec* (b) 12 *sec*

(c) 10 *sec* (d) 8 *sec*

14. A boat crosses a river with a velocity of 8 *km/h*. If the resulting velocity of boat is 10 *km/h* then the velocity of river water is

(a) 4 *km/h* (b) 6 *km/h*

(c) 8 *km/h* (d) 10 *km/h*

Critical Thinking

Objective Questions

1. If a vector $\vec{P}$ making angles α, β, and γ respectively with the *X*, *Y* and *Z* axes respectively.

Then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$

(a) 0 (b) 1

(c) 2 (d) 3

2. If the resultant of *n* forces of different magnitudes acting at a point is zero, then the minimum value of *n* is [SCRA 2000]

(a) 1 (b) 2

(c) 3 (d) 4

3. Can the resultant of 2 vectors be zero [IIIT 2000]

(a) Yes, when the 2 vectors are same in magnitude and direction

(b) No

(c) Yes, when the 2 vectors are same in magnitude but opposite in sense

(d) Yes, when the 2 vectors are same in magnitude making an angle of $\frac{2\pi}{3}$ with each other

4. The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces [Roorkee 199

(a) 12, 5 (b) 14, 4

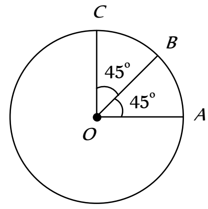
(c) 5, 13 (d) 10, 8

5. A vector $\vec{a}$ is turned without a change in its length through a small angle $d\theta$. The value of $|\Delta\vec{a}|$ and Δa are respectively

(a) $0, a\,d\theta$ (b) $a\,d\theta, 0$

(c) 0, 0 (d) None of these

6. Find the resultant of three vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in the following figure. Radius of the circle is *R*.



(a) $2R$

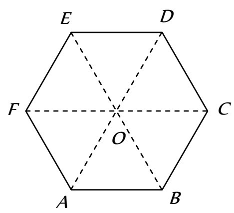
(b) $R(1+\sqrt{2})$

(c) $R\sqrt{2}$

[Manipal 2002]

(d) $R(\sqrt{2}-1)$

7. Figure shows *ABCDEF* as a regular hexagon. What is the value of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$



(a) $\overrightarrow{AO}$ [CPMT 2001]

(b) $2\overrightarrow{AO}$

(c) $4\overrightarrow{AO}$

(d) $6\overrightarrow{AO}$

8. The length of second's hand in watch is 1 *cm*. The change in velocity of its tip in 15 seconds is [MP PMT 1987]

(a) Zero (b) $\frac{\pi}{30\sqrt{2}}\,cm/\sec$

(c) $\frac{\pi}{30}\,cm/\sec$ (d) $\frac{\pi\sqrt{2}}{30}\,cm/\sec$

9. A particle moves towards east with velocity 5 *m/s*. After 10 seconds its direction changes towards north with same velocity. The average acceleration of the particle is

[CPMT 1997; IIT-JEE 1982]

(a) Zero
(b) $\frac{1}{\sqrt{2}} m/s^2 \; N-W$
(c) $\frac{1}{\sqrt{2}} m/s^2 \; N-E$
(d) $\frac{1}{\sqrt{2}} m/s^2 \; S-W$

10. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x- axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the forces $\vec{F}$ on the particle is [IIT-JEE 1998]

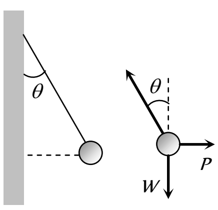
(a) $-2Ka^2$
(b) $2Ka^2$
(c) $-Ka^2$
(d) Ka^2

11. The vectors from origin to the points A and B are $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB be

(a) $\frac{5}{2}\sqrt{17}$ *sq.unit*
(b) $\frac{2}{5}\sqrt{17}$ *sq.unit*
(c) $\frac{3}{5}\sqrt{17}$ *sq.unit*
(d) $\frac{5}{3}\sqrt{17}$ *sq.unit*

12. A metal sphere is hung by a string fixed to a wall. The sphere is pushed away from the wall by a stick. The forces acting on the sphere are shown in the second diagram. Which of the following statements is wrong

(a) $P = W\tan\theta$
(b) $\vec{T} + \vec{P} + \vec{W} = 0$
(c) $T^2 = P^2 + W^2$
(d) $T = P + W$



13. The speed of a boat is 5 *km/h* in still water. It crosses a river of width 1 *km* along the shortest possible path in 15 minutes. The velocity of the river water is [IIT 1988; CBSE PMT 1998, 2000]

(a) 1 *km/h*
(b) 3 *km/h*
(c) 4 km/h
(d) 5 *km/h*

14. A man crosses a 320 *m* wide river perpendicular to the current in 4 minutes. If in still water he can swim with a speed 5/3 times that of the current, then the speed of the current, in *m/min* is

(a) 30
(b) 40
(c) 50
(d) 60.

Assertion & Reason

For AIIMS Aspirants

Read the assertion and reason carefully to mark the correct option out of the options given below:

(a) *If both assertion and reason are true and the reason is the correct explanation of the assertion.*
(b) *If both assertion and reason are true but reason is not the correct explanation of the assertion.*
(c) *If assertion is true but reason is false.*
(d) *If the assertion and reason both are false.*
(e) *If assertion is false but reason is true.*

1. Assertion : $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A} + \vec{B}$ as well as $\vec{A} - \vec{B}$.
Reason : $\vec{A} + \vec{B}$ as well as $\vec{A} - \vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A} \times \vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

2. Assertion : Angle between $\hat{i} + \hat{j}$ and $\hat{i}$ is 45°
Reason : $\hat{i} + \hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90°

3. Assertion : If θ be the angle between $\vec{A}$ and $\vec{B}$, then $\tan\theta = \frac{\vec{A} \times \vec{B}}{\vec{A}.\vec{B}}$
Reason : $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}.\vec{B}$

4. Assertion : If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is 90°
Reason : $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

5. Assertion : Vector product of two vectors is an axial vector
Reason : If $\vec{v}$ = instantaneous velocity, $\vec{r}$ = radius vector and $\vec{\omega}$ = angular velocity, then $\vec{\omega} = \vec{v} \times \vec{r}$.

6. Assertion : Minimum number of non-equal vectors in a plane required to give zero resultant is three.
Reason : If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then they must lie in one plane

7. Assertion : Relative velocity of A *w.r.t.* B is greater than the velocity of either, when they are moving in opposite directions.
Reason : Relative velocity of A *w.r.t.* $B = \vec{v}_A - \vec{v}_B$

8. Assertion : Vector addition of two vectors $\vec{A}$ and $\vec{B}$ is commutative.
Reason : $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

9. Assertion : $\vec{A}.\vec{B} = \vec{B}.\vec{A}$
Reason : Dot product of two vectors is commutative.

10. Assertion : $\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{\tau} \neq \vec{F} \times \vec{r}$
Reason : Cross product of vectors is commutative.

11. Assertion : A negative acceleration of a body is associated with a slowing down of a body. [Roorkee 1998]
Reason : Acceleration is vector quantity.

12. Assertion : A physical quantity cannot be called as a vector if its magnitude is zero.
Reason : A vector has both, magnitude and direction.

13. Assertion : The sum of two vectors can be zero.
Reason : The vector cancel each other, when they are equal and opposite.

14. Assertion : Two vectors are said to be like vectors if they have same direction but different magnitude.
Reason : Vector quantities do not have specific direction.

15. Assertion : The scalar product of two vectors can be zero.
Reason : If two vectors are perpendicular to each other, their scalar product will be zero.

16. Assertion : Multiplying any vector by an scalar is a meaningful operations.
Reason : In uniform motion speed remains constant.

17. Assertion : A null vector is a vector whose magnitude is zero and direction is arbitrary.

Reason : A null vector does not exist.

18. Assertion : If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector.

Reason : Null vector is a vector with zero magnitude.

19. Assertion : The cross product of a vector with itself is a null vector.

Reason : The cross-product of two vectors results in a vector quantity.

20. Assertion : The minimum number of non coplanar vectors whose sum can be zero, is four.

Reason : The resultant of two vectors of unequal magnitude can be zero.

21. Assertion : If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$, then $\vec{A}$ may not always be equal to $\vec{C}$

Reason : The dot product of two vectors involves cosine of the angle between the two vectors.

22. Assertion : Vector addition is commutative.

Reason : $(\vec{A}+\vec{B}) \neq (\vec{B}+\vec{A})$.

Answers

Fundamentals of Vectors

1	d	2	b	3	c	4	d	5	d
6	a	7	a	8	b	9	b	10	d
11	d	12	d	13	a	14	b	15	c
16	c	17	a	18	b	19	c	20	c
21	d	22	d	23	b	24	d	25	b
26	b	27	a	28	a	29	a	30	d
31	a	32	b	33	a	34	a		

Addition and Subtraction of Vectors

1	a	2	b	3	d	4	b	5	b
6	a	7	b	8	a	9	d	10	b
11	d	12	c	13	a	14	c	15	c
16	c	17	c	18	c	19	c	20	b
21	a	22	d	23	d	24	a	25	c
26	b	27	b	28	a	29	b	30	a
31	c	32	c	33	c	34	d	35	a
36	c	37	d	38	a	39	c	40	d
41	a	42	b	43	d	44	d	45	a
46	c	47	d	48	a	49	a	50	c
51	c	52	a	53	d				

Multiplication of Vectors

1	c	2	b	3	d	4	a	5	a
6	b	7	c	8	b	9	b	10	d
11	b	12	d	13	c	14	d	15	c
16	c	17	b	18	c	19	b	20	a
21	a	22	c	23	a	24	b	25	c
26	d	27	d	28	b	29	b	30	b
31	d	32	c	33	d	34	b	35	d
36	b	37	a	38	b	39	a	40	a
41	d	42	d	43	c	44	b	45	a
46	a	47	a	48	d	49	d	50	a
51	b	52	b	53	d	54	a	55	c
56	d	57	a	58	b	59	c		

Lami's Theorem

1	c	2	a	3	b	4	c	5	b

Relative Velocity

1	b	2	b	3	c	4	c	5	d
6	a	7	c	8	c	9	d	10	ac
11	b	12	b	13	d	14	b		

Critical Thinking Questions

1	c	2	c	3	c	4	c	5	b
6	b	7	d	8	d	9	b	10	c
11	a	12	d	13	b	14	d		

Assertion and Reason

1	a	2	a	3	d	4	b	5	c
6	b	7	a	8	b	9	a	10	c
11	b	12	e	13	a	14	c	15	a
16	b	17	c	18	b	19	b	20	c
21	a	22	c						

Answers and Solutions

Fundamentals of Vectors

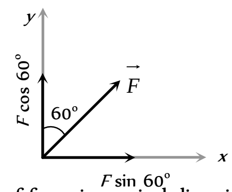
1. (d) As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in XZ plane and projection of this vector on y-axis is zero.

2. (b) If a point have coordinate (x, y, z) then its position vector $= x\hat{i} + y\hat{j} + z\hat{k}$.

3. (c) Displacement vector $\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

$= (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k} = \hat{i} + \hat{j}$

4. (d)



The component of force in vertical direction

$= F\cos\theta = F\cos 60° = 5 \times \frac{1}{2} = 2.5\ N$

5. (d) $|B| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

Unit vector in the direction of A will be $\hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

So required vector $= 25\left(\frac{3\hat{i} + 4\hat{j}}{5}\right) = 15\hat{i} + 20\hat{j}$

6. (a) Let the components of $\vec{A}$ makes angles α, β and γ with x, y and z axis respectively then $\alpha = \beta = \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \dfrac{1}{\sqrt{3}}$

$\therefore A_x = A_y = A_z = A\cos\alpha = \dfrac{A}{\sqrt{3}}$

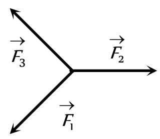
7. (a) $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k} \therefore |\vec{A}| = \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{45}$

$\therefore \cos\alpha = \dfrac{2}{\sqrt{45}}, \quad \cos\beta = \dfrac{4}{\sqrt{45}}, \quad \cos\gamma = \dfrac{-5}{\sqrt{45}}$

8. (b) Unit vector along y axis $= \hat{j}$ so the required vector $= \hat{j} - [(\hat{i} - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} - 7\hat{k})] = -4\hat{i} - 2\hat{j} + 5\hat{k}$

9. (b) $\vec{F}_3 = \vec{F}_1 + \vec{F}_2$

There should be minimum three coplaner vectors having different magnitude which should be added to give zero resultant

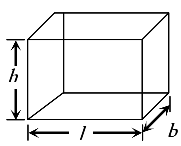


10. (d) Diagonal of the hall $= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{10^2 + 12^2 + 14^2}$

$= \sqrt{100 + 144 + 196}$

$= \sqrt{400} = 20m$



11. (d) Total angle $= 100 \times \dfrac{\pi}{50} = 2\pi$

So all the force will pass through one point and all forces will be balanced. *i.e.* their resultant will be zero.

12. (d) $\vec{r} = \vec{r}_2 - \vec{r}_1 = (-2\hat{i} - 2\hat{j} + 0\hat{k}) - (4\hat{i} - 4\hat{j} + 0\hat{k})$

$\Rightarrow \vec{r} = -6\hat{i} + 2\hat{j} + 0\hat{k}$

$\therefore |\vec{r}| = \sqrt{(-6)^2 + (2)^2 + 0^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

13. (a) $\vec{P} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j} \therefore |\vec{P}| = \sqrt{\left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2} = 1$

$\therefore$ It is a unit vector.

14. (b)

15. (c) $\hat{R} = \dfrac{\vec{R}}{|R|} = \dfrac{\hat{i} + \hat{j}}{\sqrt{1^2 + 1^2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

16. (c) $\vec{R} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Length in XY plane $= \sqrt{R_x^2 + R_y^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$

17. (a) If the angle between all forces which are equal and lying in one plane are equal then resultant force will be zero.

18. (b) $\vec{A} = \hat{i} + \hat{j} \Rightarrow |A| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos\alpha = \dfrac{A_x}{|A|} = \dfrac{1}{\sqrt{2}} = \cos 45° \therefore \alpha = 45°$

19. (c)

20. (c)

21. (d) All quantities are tensors.

22. (d) $\vec{P} + \vec{Q} = P\hat{P} + Q\hat{Q}$

23. (b) $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = -a\omega\sin\omega t\,\hat{i} + a\omega\cos\omega t\,\hat{j}$

As $\vec{r}.\vec{v} = 0$ therefore velocity of the particle is perpendicular to the position vector.

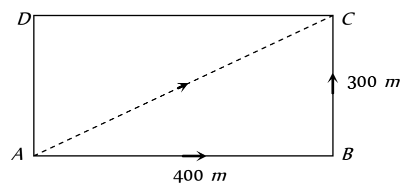
24. (d) Displacement, electrical and acceleration are vector quantities.

25. (b) Magnitude of unit vector = 1

$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

By solving we get $c = \sqrt{0.11}$

26. (b)



Displacement $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(400)^2 + (300)^2} = 500m$

Distance $= AB + BC = 400 + 300 = 700m$

27. (a) Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\hat{R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

28. (a) $\phi = \vec{B}.\vec{A}$. In this formula $\vec{A}$ is a area vector.

29. (a) $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k} = \hat{i} + \hat{j} - \hat{k}$

$\hat{r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

30. (d) $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|A||B|} = \dfrac{9 + 16 + 25}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}} = \dfrac{50}{50} = 1$

$\Rightarrow \cos\theta = 1 \therefore \theta = \cos^{-1}(1)$

31. (a) $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$

at $t = 0$, $\vec{r}_1 = 7\hat{k}$

at $t = 10$ sec, $\vec{r}_2 = 300\hat{i} + 400\hat{j} + 7\hat{k}$,

$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1 = 300\hat{i} + 400\hat{j}$

$|\overrightarrow{\Delta r}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(300)^2 + (400)^2} = 500m$

32. (b) Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 8\hat{i} + 8\hat{j} = 12\hat{i} + 5\hat{j}$

$\hat{R} = \dfrac{\vec{R}}{|R|} = \dfrac{12\hat{i} + 5\hat{j}}{\sqrt{(12)^2 + (5)^2}} = \dfrac{12\hat{i} + 5\hat{j}}{13}$

33. (a) $\frac{\vec{A}.\vec{B}}{|\hat{i}+\hat{j}|} = \frac{(2\hat{i}+3\hat{j})(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

34. (a) $\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{(3\hat{i}+4\hat{j}+5\hat{k})(3\hat{i}+4\hat{j}-5\hat{k})}{\sqrt{9+16+25}\sqrt{9+16+25}}$

$= \frac{9+16-25}{50} = 0$

$\Rightarrow \cos\theta = 0$, $\therefore \theta = 90°$

Addition and Subtraction of Vectors

1. (a) For 17 *N* both the vector should be parallel *i.e.* angle between them should be zero.

For 7 *N* both the vectors should be antiparallel *i.e.* angle between them should be 180°

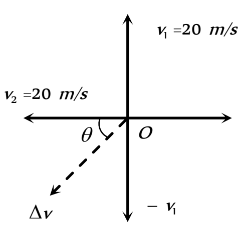
For 13 *N* both the vectors should be perpendicular to each other *i.e.* angle between them should be 90°

2. (b) $\vec{A}+\vec{B} = 4\hat{i}-3\hat{j}+6\hat{i}+8\hat{j} = 10\hat{i}+5\hat{j}$

$|\vec{A}+\vec{B}| = \sqrt{(10)^2+(5)^2} = 5\sqrt{5}$

$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$

3. (d) From figure



$\vec{v}_1 = 20\hat{j}$ and $\vec{v}_2 = -20\hat{i}$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = -20(\hat{i}+\hat{j})$

$|\Delta\vec{v}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45°$ *i.e.* *S–W*

4. (b) Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is

$\vec{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$

$= 1+1+2\cos\theta$

Since it is given that n_s is also a unit vector, therefore

$1 = 1+1+2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2}$ $\therefore \theta = 120°$

Now the difference vector is $\hat{n}_d = \hat{n}_1 - \hat{n}_2$ or

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1+1-2\cos(120°)$

$\therefore n_d^2 = 2-2(-1/2) = 2+1 = 3 \Rightarrow n_d = \sqrt{3}$

5. (b) $\vec{A}-2\vec{B}+3\vec{C} = (2\hat{i}+\hat{j}) - 2(3\hat{j}-\hat{k}) + 3(6\hat{i}-2\hat{k})$

$= 2\hat{i}+\hat{j}-6\hat{j}+2\hat{k}+18\hat{i}-6\hat{k} = 20\hat{i}-5\hat{j}-4\hat{k}$

6. (a) $\vec{P}_1 = mv\sin\theta\hat{i} - mv\cos\theta\hat{j}$

and $\vec{P}_2 = mv\sin\theta\hat{i} + mv\cos\theta\hat{j}$

So change in momentum

$\Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = 2mv\cos\theta\hat{j}$, $|\Delta\vec{P}| = 2mv\cos\theta$

7. (b) $R = \sqrt{A^2+B^2+2AB\cos\theta}$

By substituting, $A = F$, $B = F$ and $R = F$ we get

$\cos\theta = \frac{1}{2}$ $\therefore \theta = 120°$

8. (a)

9. (d) If two vectors $\vec{A}$ and $\vec{B}$ are given then the resultant R_{max} = $A+B = 7N$ and $R_{min} = 4-3 = 1N$

i.e. net force on the particle is between 1 *N* and 7 *N*.

10. (b) If $\vec{C}$ lies outside the plane then resultant force can not be zero.

11. (d)

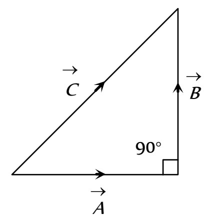
12. (c) $F = \sqrt{F_1^2+F_2^2+2F_1F_2\cos 90°} = \sqrt{F_1^2+F_2^2}$

13. (a)

14. (c)

15. (c) $C = \sqrt{A^2+B^2}$

The angle between *A* and *B* is $\frac{\pi}{2}$



16. (c) $\vec{R} = \vec{A}+\vec{B} = 6\hat{i}+7\hat{j}+3\hat{i}+4\hat{j} = 9\hat{i}+11\hat{j}$

$\therefore |\vec{R}| = \sqrt{9^2+11^2} = \sqrt{81+121} = \sqrt{202}$

17. (c) $R = \sqrt{12^2+5^2+6^2} = \sqrt{144+25+36} = \sqrt{205} = 14.31\,m$

18. (c) $\vec{A} = 3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B} = \hat{i}-3\hat{j}+5\hat{k}$, $\vec{C} = 2\hat{i}-\hat{j}+4\hat{k}$

$|\vec{A}| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{9+4+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{1^2+(-3)^2+5^2} = \sqrt{1+9+25} = \sqrt{35}$

$|\vec{A}| = \sqrt{2^2+1^2+(-4)^2} = \sqrt{4+1+16} = \sqrt{21}$

As $B = \sqrt{A^2+C^2}$ therefore *ABC* will be right angled triangle.

19. (c)

20. (b) $\vec{C}+\vec{A} = \vec{B}$.

The value of *C* lies between $A-B$ and $A+B$

$\therefore |\vec{C}| < |\vec{A}|$ or $|\vec{C}| < |\vec{B}|$

21. (a)

22. (d)

23. (d) Here all the three force will not keep the particle in equilibrium so the net force will not be zero and the particle will move with an acceleration.

24. (a) $A+B = 16$ (given) ...(i)

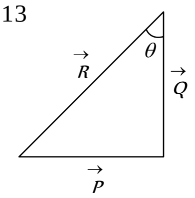
$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \tan 90°$

$\therefore A+B\cos\theta = 0 \Rightarrow \cos\theta = \frac{-A}{B}$ ...(ii)

$8 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ...(iii)

By solving eq. (i), (ii) and (iii) we get $A = 6N,\ \ B = 10N$

25. (c) $|\vec{P}| = 5,\ |\vec{Q}| = 12$ and $|\vec{R}| = 13$



$\cos\theta = \frac{Q}{R} = \frac{12}{13}$

$\therefore\ \theta = \cos^{-1}\left(\frac{12}{13}\right)$

26. (b) $\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ...(i)

$\therefore\ \tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$\therefore\ \cos\theta = -\frac{A}{B}$

Hence, from (i) $\frac{B^2}{4} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3}\,\frac{B}{2}$

$\Rightarrow \cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2}\ \therefore\ \theta = 150°$

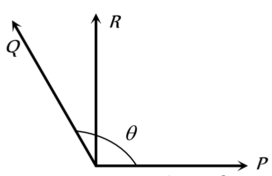
27. (b) $(\hat{i} - 2\hat{j} + 2\hat{k}) + (2\hat{i} + \hat{j} - \hat{k}) + \vec{R} = i$

$\therefore$ Required vector $\vec{R} = -2\hat{i} + \hat{j} - \hat{k}$

28. (a) Resultant $\vec{R} = \vec{P} + \vec{Q} + \vec{P} - \vec{Q} = 2\vec{P}$

The angle between $\vec{P}$ and $2\vec{P}$ is zero.

29. (b)



$\Rightarrow \tan 90° = \frac{Q\sin\theta}{P + Q\cos\theta} \Rightarrow P + Q\cos\theta = 0$

$\cos\theta = \frac{-P}{Q}\ \therefore\ \theta = \cos^{-1}\left(\frac{-P}{Q}\right)$

30. (a) According to problem $P + Q = 3$ and $P - Q = 1$

By solving we get $P = 2$ and $Q = 1\ \therefore\ \frac{P}{Q} = 2 \Rightarrow P = 2Q$

31. (c)

32. (c)

33. (c)

34. (d) $F_1 + F_2 + F_3 = 0 \Rightarrow 4\hat{i} + 6\hat{j} + F_3 = 0$

$\therefore\ \vec{F}_3 = -4\hat{i} - 6\hat{j}$

35. (a) $\Delta v = 2v\sin\left(\frac{\theta}{2}\right) = 2 \times v \times \sin 90°$

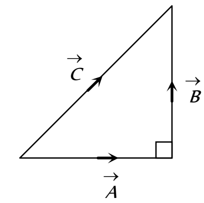
$= 2 \times 100 = 200\ km/hr$

36. (c)

37. (d) Resultant velocity $= \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225} = \sqrt{625} = 25\ km/hr$

38. (a) $C = \sqrt{A^2 + B^2}$

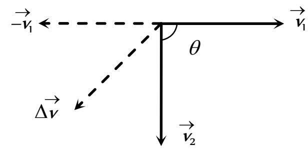


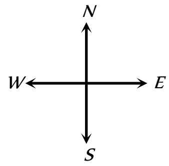
$= \sqrt{3^2 + 4^2} = 5$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$

39. (c)

40. (d)





If the magnitude of vector remains same, only direction change by θ then

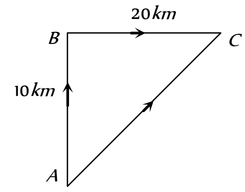
$\overrightarrow{\Delta v} = \overrightarrow{v_2} - \overrightarrow{v_1}$, $\overrightarrow{\Delta v} = \overrightarrow{v_2} + (\overrightarrow{-v_1})$

Magnitude of change in vector $|\overrightarrow{\Delta v}| = 2v\sin\left(\frac{\theta}{2}\right)$

$|\overrightarrow{\Delta v}| = 2 \times 10 \times \sin\left(\frac{90°}{2}\right) = 10\sqrt{2} = 14.14\ m/s$

Direction is south-west as shown in figure.

41. (a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$



$AC = \sqrt{(AB)^2 + (BC)^2}$

$= \sqrt{(10)^2 + (20)^2}$

$= \sqrt{100 + 400} = \sqrt{500} = 22.36\ km$

42. (b) $\cos\theta = \frac{\vec{F}_1 . \vec{F}_2}{|F_1||F_2|}$

$= \frac{(5\hat{i} + 10\hat{j} - 20\hat{k}).(10\hat{i} - 5\hat{j} - 15\hat{k})}{\sqrt{25 + 100 + 400}\sqrt{100 + 25 + 225}} = \frac{50 - 50 + 300}{\sqrt{525}\sqrt{350}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}\ \therefore\ \theta = 45°$

43. (d) If two vectors A and B are given then Range of their resultant can be written as $(A - B) \le R \le (A + B)$.

i.e. $R_{max} = A + B$ and $R_{min} = A - B$

If $B = 1$ and $A = 4$ then their resultant will lies in between $3N$ and $5N$. It can never be $2N$.

44. (d) $A = 3N,\ B = 2N$ then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

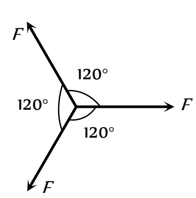
$R = \sqrt{9 + 4 + 12\cos\theta}$ ...(i)

Now $A = 6N,\ B = 2N$ then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ ...(ii)

from (i) and (ii) we get $\cos\theta = -\frac{1}{2}\ \therefore\ \theta = 120°$

45. (a) In N forces of equal magnitude works on a single point and their resultant is



zero then angle between any two forces is given $\theta = \frac{360}{N} = \frac{360}{3} = 120°$

If these three vectors are represented by three sides of triangle then they form equilateral triangle

46. (c) Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by $\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $\theta = 0°$ then $|\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$

47. (d) $R_{\max} = A + B = 17$ when $\theta = 0°$

$R_{\min} = A - B = 7$ when $\theta = 180°$

by solving we get $A = 12$ and $B = 5$

Now when $\theta = 90°$ then $R = \sqrt{A^2 + B^2}$

$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$

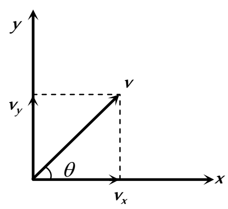
48. (a) If two vectors are perpendicular then their dot product must be equal to zero. According to problem

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0 \Rightarrow \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$

$\Rightarrow A^2 - B^2 = 0 \Rightarrow A^2 = B^2$

$\therefore$ $A = B$ *i.e.* two vectors are equal to each other in magnitude.

49. (a) $v_y = 20$ and $v_x = 10$



$\therefore$ velocity $\vec{v} = 10\hat{i} + 20\hat{j}$

direction of velocity with x axis

$\tan\theta = \frac{v_y}{v_x} = \frac{20}{10} = 2$

$\therefore$ $\theta = \tan^{-1}(2)$

50. (c) $R_{\max} = A + B$ when $\theta = 0°$ $\therefore$ $R_{\max} = 12 + 8 = 20N$

51. (c) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $A = B = P$ and $\theta = 120°$ then $R = P$

52. (a) Sum of the vectors $\vec{R} = 5\hat{i} + 8\hat{j} + 2\hat{i} + 7\hat{j} = 7\hat{i} + 15\hat{j}$

magnitude of $\vec{R} = |\vec{R}| = \sqrt{49 + 225} = \sqrt{274}$

53. (d)

Multiplication of Vectors

1. (c) Given vectors can be rewritten as $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore$ $\vec{A}.\vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4 \Rightarrow \alpha = -1/2$

2. (b) Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ *i.e.* $\frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \Rightarrow \lambda = 2$.

3. (d) $W = \vec{F}.\vec{S} = FS\cos\theta$

$= 50 \times 10 \times \cos 60° = 50 \times 10 \times \frac{1}{2} = 250\ J.$

4. (a) $S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4 \times 11 + 1 \times 11 + 3 \times 15) = 100\ J.$

5. (a) $(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$. Thus

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

or $A^2 + \vec{B}.\vec{A} - \vec{A}.\vec{B} - B^2 = 0$

Because of commutative property of dot product $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

$\therefore$ $A^2 - B^2 = 0$ or $A = B$

Thus the ratio of magnitudes $A/B = 1$

6. (b) Let $\vec{A}.(\vec{B} \times \vec{A}) = \vec{A}.\vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ Which is perpendicular to both vector $\vec{A}$ and $\vec{B}$ $\therefore$ $\vec{A}.\vec{C} = 0$

7. (c) We know that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ because the angle between these two is always 90°.

But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or π. Then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = 0$.

8. (b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= (1 \times 4 - 2 \times -2)\hat{i} + (2 \times 2 - 4 \times 3)\hat{j} + (3 \times -2 - 1 \times 2)\hat{k}$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\therefore$ Magnitude of $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2}$

$= 8\sqrt{3}$

9. (b) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= [(2 \times 4) - (3 \times -3)]\hat{i} + [(2 \times 3) - (3 \times 4)]\hat{j}$

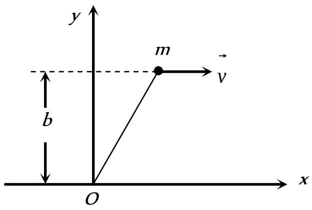
$+ [(3 \times -3) - (2 \times 2)]\hat{k} = 17\hat{i} - 6\hat{j} - 13\hat{k}$

10. (d) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ vector also, must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ also but the cross product $(\vec{A}\times\vec{B})$ gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

11. (b) We know that, Angular momentum

$\vec{L}=\vec{r}\times\vec{p}$ in terms of component becomes

$$\vec{L}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}$$



As motion is in *x-y* plane ($z=0$ and $P_z=0$), so

$\vec{L}=\vec{k}(xp_y-yp_x)$

Here $x=vt$, $y=b$, $p_x=mv$ and $p_y=0$

$\therefore\ \vec{L}=\vec{k}\left[vt\times 0-b\,mv\right]=-mvb\,\hat{k}$

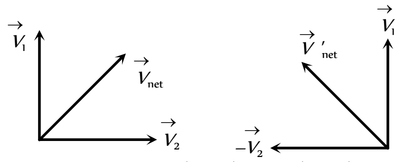
12. (d) $\vec{F_1}.\vec{F_2}=(2\hat{j}+5\hat{k})(3\hat{j}+4\hat{k})$

$=6+20=20+6=26$

13. (c) Force F lie in the *x-y* plane so a vector along *z*-axis will be perpendicular to *F*.

14. (d) $\vec{A}.\vec{B}=|\vec{A}|.|\vec{B}|.\cos\theta=\vec{A}.\vec{B}.\cos 90^\circ=0$

15. (c)



According to problem $|\vec{V_1}+\vec{V_2}|=|\vec{V_1}-\vec{V_2}|$

$\Rightarrow|\vec{V}_{net}|=|\vec{V}'_{net}|$

So V_1 and V_2 will be mutually perpendicular.

16. (c) $W=\vec{F}.\vec{r}=(5\hat{i}+3\hat{j})(2\hat{i}-\hat{j})=10-3=7\,J$.

17. (b) $\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{-2+6-4}{\sqrt{14}\sqrt{21}}=0\ \therefore\ \theta=90^\circ$

18. (c) $(\hat{i}+\hat{j}).(\hat{j}+\hat{k})=0+0+1+0=1$

$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{1}{\sqrt{2}\times\sqrt{2}}=\frac{1}{2}\ \therefore\ \theta=60^\circ$

19. (b) $P=\vec{F}.\vec{v}=20\times 6+15\times(-4)+(-5)\times 3$

$=120-60-15=120-75=45\,J/s$

20. (a) $\cos\theta=\frac{\vec{P}.\vec{Q}}{PQ}=1\ \therefore\ \theta=0^\circ$

21. (a) $W=\vec{F}.\vec{s}=(5\hat{i}+6\hat{j}+4\hat{k})(6\hat{i}-5\hat{k})=30-20=10\,J$

22. (c) $\vec{A}.\vec{B}=0\ \therefore\ \theta=90^\circ$

23. (a) $\vec{P}.\vec{Q}=0\ \therefore\ a^2-2a-3=0\Rightarrow a=3$

24. (b) $W=\vec{F}.\vec{r}=(-2\hat{i}+15\hat{j}+6\hat{k})(10\hat{j})=150$

25. (c) $P_x=2\cos t$, $P_y=2\sin t\ \therefore\ \vec{P}=2\cos t\,\hat{i}+2\sin t\,\hat{j}$

$\vec{F}=\frac{d\vec{P}}{dt}=-2\sin t\,\hat{i}+2\cos t\,\hat{j}$

$\vec{F}.\vec{P}=0\ \therefore\ \theta=90^\circ$

26. (d) $|\vec{A}\times\vec{B}|=|(2\hat{i}+3\hat{j})\times(\hat{i}+4\hat{j})|=|5\hat{k}|=5\ units$

27. (d)

28. (b) $\vec{A}\times\vec{B}=0\ \therefore\ \sin\theta=0\ \therefore\ \theta=0^\circ$

Two vectors will be parallel to each other.

29. (b) $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ are parallel and opposite to each other. So the angle will be π.

30. (b) Vector $(\vec{P}+\vec{Q})$ lies in a plane and vector $(\vec{P}\times\vec{Q})$ is perpendicular to this plane *i.e.* the angle between given vectors is $\frac{\pi}{2}$.

31. (d) $\sqrt{2^2+3^2+2\times 2\times 3\times\cos\theta}=1$

By solving we get $\theta=180^\circ\ \therefore\ \vec{A}\times\vec{B}=0$

32. (c) Dot product of two perpendicular vector will be zero.

33. (d) $\cos\theta=\frac{\vec{A}\vec{B}}{AB}=\frac{42+24-12}{\sqrt{36+36+9}\sqrt{49+16+16}}=\frac{56}{9\sqrt{71}}$

$\cos\theta=\frac{56}{9\sqrt{71}}\ \therefore\ \sin\theta=\frac{\sqrt{5}}{3}$ or $\theta=\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

34. (b) Direction of vector A is along *z*-axis $\therefore\ \vec{A}=a\hat{k}$

Direction of vector B is towards north $\therefore\ \vec{B}=b\hat{j}$

Now $\vec{A}\times\vec{B}=a\hat{k}\times b\hat{j}=ab(-\hat{j})$

$\therefore$ The direction is $\vec{A}\times\vec{B}$ is along west.

35. (d) $\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{1}{\sqrt{2}\sqrt{2}}=\frac{1}{2}\ \therefore\ \theta=60^\circ$

36. (d) $\overrightarrow{AB}=(4\hat{i}+5\hat{j}+6\hat{k})-(3\hat{i}+4\hat{j}+5\hat{k})=\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{CD}=(4\hat{i}+6\hat{j})-(7\hat{i}+9\hat{j}+3\hat{k})=-3\hat{i}-3\hat{j}-3\hat{k}$

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, because its cross-products is 0.

37. (a) $W=\vec{F}\,\vec{S}=(4\hat{i}+5\hat{j})(3\hat{i}+6\hat{j})=12$

38. (b) $|\vec{A}\times\vec{B}|=\vec{A}.\vec{B}\Rightarrow AB\sin\theta=AB\cos\theta\Rightarrow\tan\theta=1$

$\therefore\ \theta=45^\circ$

39. (a)

40. (a) $\vec{v}=\vec{\omega}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 0 & 4 & -3\end{vmatrix}=\hat{i}(6-8)-\hat{j}(-3)+4\hat{k}$

$-2\vec{i}+3\vec{j}+4\vec{k}$

$|\vec{v}|=\sqrt{(-2)^2+(3)^2+4^2}=\sqrt{29}\ unit$

41. (d) $\vec{a}.\vec{b}=0$ *i.e.* $\vec{a}$ and $\vec{b}$ will be perpendicular to each other

$\vec{a}.\vec{c}=0$ *i.e.* $\vec{a}$ and $\vec{c}$ will be perpendicular to each other

$\vec{b} \times \vec{c}$ will be a vector perpendicular to both $\vec{b}$ and $\vec{c}$

So $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$

42. (d) Area $= \left|2\hat{i} \times 2\hat{j}\right| = \left|4\hat{k}\right| = 4\ unit$

43. (c) $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{C} = \vec{A} \times \vec{B} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) \times \left(6\hat{i} - 3\hat{j} + 2\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix} = \hat{i} - 10\hat{j} - 18\hat{k}$$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

$$= \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{\sqrt{1^2 + 10^2 + 18^2}} = \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$$

44. (b) $\vec{A} = \hat{j} + 3\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -7\hat{i} + 3\hat{j} - \hat{k}$$

Hence area $= |\vec{C}| = \sqrt{49 + 9 + 1} = \sqrt{59}\ squnit$

45. (a) $$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$$

i.e. the angular momentum is perpendicular to *x*-axis.

46. (a) $\vec{A} \times \vec{B}$ is a vector perpendicular to plane $\vec{A} + \vec{B}$ and hence perpendicular to $\vec{A} + \vec{B}$.

47. (a) $\vec{\tau} = \vec{r} \times \vec{F} = (7\hat{i} + 3\hat{j} + \hat{k})(-3\hat{i} + \hat{j} + 5\hat{k})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

48. (d) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = \vec{A} \times \vec{A} - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - \vec{B} \times \vec{B}$

$= 0 - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - 0 = \vec{B} \times \vec{A} + \vec{B} \times \vec{A} = 2(\vec{B} \times \vec{A})$

49. (d) For perpendicular vector $\vec{A}.\vec{B} = 0$

$\Rightarrow (5\hat{i} + 7\hat{j} - 3\hat{k}).(2\hat{i} + 2\hat{j} - a\hat{k}) = 0$

$\Rightarrow 10 + 14 + 3a = 0 \Rightarrow a = -8$

50. (a) Mass $= \dfrac{\text{Force}}{\text{Acceleration}} = \dfrac{|\vec{F}|}{a}$

$$= \frac{\sqrt{36 + 64 + 100}}{1} = 10\sqrt{2}\ kg$$

51. (a) Area of parallelogram $= \vec{A} \times \vec{B}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = (8)\hat{i} + (8)\hat{j} - (8)\hat{k}$$

Magnitude $= \sqrt{64 + 64 + 64} = 8\sqrt{3}$

52. (b) Radius vector $\vec{r} = \vec{r_2} - \vec{r_1} = (2\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k})$

$\therefore \vec{r} = -4\hat{j}$

Linear momentum $\vec{p} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{L} = \vec{r} \times \vec{p} = (-4\hat{j}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 0 \\ 2 & 3 & -1 \end{vmatrix} = 4\hat{i} - 8\hat{k}$$

53. (d) $$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$$

54. (a)

55. (c) $\vec{A}.\vec{B} = AB\cos\theta$

In the problem $\vec{A}.\vec{B} = -AB$ i.e. $\cos\theta = -1 \therefore \theta = 180°$

i.e. $\vec{A}$ and $\vec{B}$ acts in the opposite direction.

56. (d) $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A}.\vec{B})$

$AB\sin\theta = \sqrt{3}AB\cos\theta \Rightarrow \tan\theta = \sqrt{3} \therefore \theta = 60°$

Now $|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$= \sqrt{A^2 + B^2 + 2AB\left(\frac{1}{2}\right)} = (A^2 + B^2 + AB)^{1/2}$$

57. (a) $W = \vec{F}.\vec{s} = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} - 3\hat{k}) = -12 + 2c - 6$

Work done $= 6J$ (given)

$\therefore -12 + 2c - 6 = 6 \Rightarrow c = 12$

58. (b) $W = \vec{F}.\vec{s} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = 7\ J$

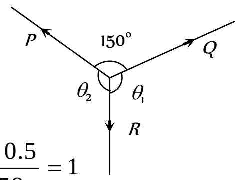
59. (c) $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

for parallel vectors $\theta = 0°$ or $180°$, $\sin\theta = 0$

$\therefore\ \vec{A}\times\vec{B}=\hat{0}$

Lami's Theorem

1. (c) $\dfrac{P}{\sin\theta_1}=\dfrac{Q}{\sin\theta_2}=\dfrac{R}{\sin 150^\circ}$



$\Rightarrow \dfrac{1.93}{\sin\theta_1}=\dfrac{R}{\sin 150^\circ}$

$\Rightarrow R=\dfrac{1.93\times\sin150^\circ}{\sin\theta_1}=\dfrac{1.93\times 0.5}{0.9659}=1$

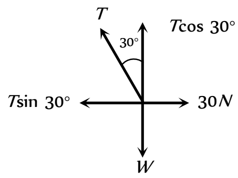
2. (a) According to Lami's theorem

$\dfrac{P}{\sin\alpha}=\dfrac{Q}{\sin\beta}=\dfrac{R}{\sin\gamma}$

3. (b)

4. (c)

5. (b)



From the figure $T\sin30^o=30$ ...(i)

$T\cos30^o=W$ ...(ii)

By solving equation (i) and (ii) we get

$W=30\sqrt{3}N$ and $T=60N$

Relative Velocity

1. (b) The two car (say *A* and *B*) are moving with same velocity, the relative velocity of one (say B) with respect to the other

$A, \vec{v}_{BA}=\vec{v}_B-\vec{v}_A=v-v=0$

So the relative separation between them (= 5 *km*) always remains the same.

Now if the velocity of car (say *C*) moving in opposite direction to *A* and *B*, is $\vec{v}_C$ relative to ground then the velocity of car *C* relative to *A* and B will be $\vec{v}_{rel.}=\vec{v}_C-\vec{v}$

But as $\vec{v}$ is opposite to v_c

So $v_{rel}=v_c-(-30)=(v_C+30)km/hr.$

So, the time taken by it to cross the cars *A* and *B*

$t=\dfrac{d}{v_{rel}}\Rightarrow\dfrac{4}{60}=\dfrac{5}{v_C+30}$

$\Rightarrow v_C=45\,km/hr.$

2. (b) When the man is at rest *w.r.t.* the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.

Here $\vec{v}_{rg}$ = velocity of rain with respect to the ground

$\vec{v}_{mg}$ = velocity of the man with respect to the ground.

and $\vec{v}_{rm}$ = velocity of the rain with respect to the man,

We have $\vec{v}_{rg}=\vec{v}_{rm}+\vec{v}_{mg}$ ......(i)

Taking horizontal components equation (i) gives

$v_{rg}\sin30^\circ=v_{mg}=10\,km/hr$

or $v_{rg}=\dfrac{10}{\sin30^\circ}=20\,km/hr$

3. (c) Taking vertical components equation (i) gives

$v_{rg}\cos30^\circ=v_{rm}=20\dfrac{\sqrt{3}}{2}=10\sqrt{3}\,km/hr$

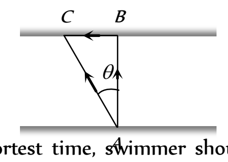
4. (c) Relative velocity = $(3\hat{i}+4\hat{j})-(-3\hat{i}-4\hat{j})=6\hat{i}+8\hat{j}$

5. (d) Relative velocity of parrot with respect to train

$=5-(-10)=5+10=15\,m/\sec$

Time taken by the parrot $=\dfrac{d}{v_{rel.}}=\dfrac{150}{15}=10\sec$.

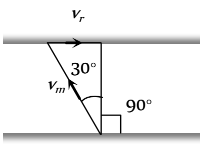
6. (a)



For shortest time, swimmer should swim along *AB*, so he will reach at point *C* due to the velocity of river.

i.e. he should swim due north.

7. (c)



$\sin30^o=\dfrac{v_r}{v_m}=\dfrac{1}{2}\Rightarrow v_r=\dfrac{v_m}{2}=\dfrac{0.5}{2}=0.25\,m/s$

8. (c) $\vec{v}_B+\vec{v}_A=\vec{v}_B+\vec{v}_A=80+65=145\,km/hr$

9. (d) Relative speed of police with respect to thief

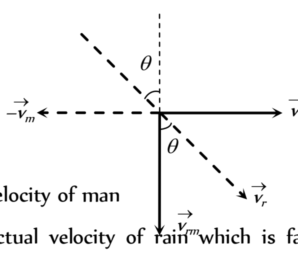
$=10-9=1\,m/s$

Instantaneous separation = 100 *m*

Time = $\dfrac{\text{distance}}{\text{veclotiy}}=\dfrac{100}{1}=100\sec$.

10. (a,c)

11. (b) A man is sitting in a bus and travelling from west to east, and the rain drops are appears falling vertically down.



v_m = velocity of man

v_r = Actual velocity of rain which is falling at an angle θ with vertical

v_{rm} = velocity of rain *w.r.t.* to moving man

If the another man observe the rain then he will find that actually rain falling with velocity v_r at an angle going from west to east.

12. (b) Boat covers distance of 16 *km* in a still water in 2 hours.

i.e. $v_B = \frac{16}{2} = 8\ km/hr$

Now velocity of water $\Rightarrow v_w = 4 km/hr$.

Time taken for going upstream

$$t_1 = \frac{8}{v_B - v_w} = \frac{8}{8-4} = 2hr$$

(As water current oppose the motion of boat)

Time taken for going down stream

$$t_2 = \frac{8}{v_B + v_w} = \frac{8}{8+4} = \frac{8}{12} hr$$

(As water current helps the motion of boat)

$\therefore$ Total time $= t_1 + t_2 = \left(2 + \frac{8}{12}\right) hr$ or $2hr\ 40 min$

13. (d) Relative velocity $= 10 + 5 = 15\ m/s$.

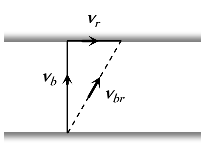
Time taken by the bird to cross the train $= \frac{120}{15} = 8$ sec

14. (b) $\overrightarrow{v_{br}} = \overrightarrow{v_b} + \overrightarrow{v_r}$

$\Rightarrow v_{br} = \sqrt{v_b^2 + v_r^2}$

$\Rightarrow 10 = \sqrt{8^2 + v_r^2}$

$\Rightarrow v_r = 6 km/hr$.



Critical Thinking Questions

1. (c) $\sin^2\alpha + \sin^2\beta + \sin\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

2. (c) If vectors are of equal magnitude then two vectors can give zero resultant, if they works in opposite direction. But if the vectors are of different magnitudes then minimum three vectors are required to give zero resultant.

3. (c)

4. (c) Let P be the smaller force and Q be the greater force then according to problem –

$P + Q = 18$ ......(i)

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12$ ....(ii)

$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90 = \infty$

$\therefore\ P + Q\cos\theta = 0$ ....(iii)

By solving (i), (ii) and (iii) we will get $P = 5$, and $Q = 13$

5. (b) From the figure $|\overrightarrow{OA}| = a$ and $|\overrightarrow{OB}| = a$

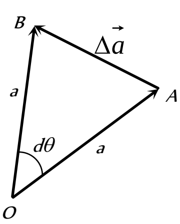
Also from triangle rule $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \Delta\vec{a}$

$\Rightarrow |\Delta\vec{a}| = AB$

Using angle $= \frac{\text{arc}}{\text{radius}}$

$\Rightarrow\ AB = a\,.\,d\theta$

So $|\Delta\vec{a}| = a\,d\theta$



Δa means change in magnitude of vector *i.e.* $|\overrightarrow{OB}| - |\overrightarrow{OA}|$

$\Rightarrow\ a - a = 0$

So $\Delta a = 0$

6. (b) $R_{net} = R + \sqrt{R^2 + R^2} = R + \sqrt{2}R = R(\sqrt{2} + 1)$

7. (d)

8. (d) $\Delta v = 2v\sin\left(\frac{90°}{2}\right) = 2v\sin 45° = 2v \times \frac{1}{\sqrt{2}} = \sqrt{2}v$

$= \sqrt{2} \times r\omega = \sqrt{2} \times 1 \times \frac{2\pi}{60} = \frac{\sqrt{2}\pi}{`30}\ cm/s$

9. (b) $\Delta v = 2v\sin\left(\frac{\theta}{2}\right) = 2 \times 5 \times \sin 45° = \frac{10}{\sqrt{2}}$

$\therefore\ a = \frac{\Delta v}{\Delta t} = \frac{10/\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\ m/s^2$

10. (c) For motion of the particle from (0, 0) to (*a*, 0)

$\vec{F} = -K(0\hat{i} + a\hat{j}) \Rightarrow\ \vec{F} = -Ka\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from (0, 0) to (*a*, 0) is given by

$W = \vec{F}.\vec{r} = -Ka\hat{j}.a\hat{i} = 0$

For motion (*a*, 0) to (*a*, *a*)

$\vec{F} = -K(a\hat{i} + a\hat{j})$ and displacement

$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from (*a*, 0) to (*a*, *a*) $W = \vec{F}.\vec{r}$

$= -K(a\hat{i} + a\hat{j}).a\hat{j} = -Ka^2$

So total work done $= -Ka^2$

11. (a) Given $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\overrightarrow{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore\ (\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

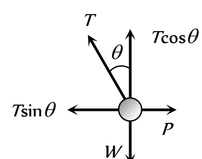
$= (12 - 2)\hat{i} + (4 + 6)\hat{j} + (3 + 12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k} \Rightarrow |\vec{a} \times \vec{b}| = \sqrt{10^2 + 10^2 + 15^2}$

$= \sqrt{425}\ = 5\sqrt{17}$

Area of $\Delta OAB = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{5\sqrt{17}}{2}$ *sq.unit.*

12. (d)



As the metal sphere is in equilibrium under the effect of three forces therefore $\vec{T} + \vec{P} + \vec{W} = 0$

From the figure $T\cos\theta = W$ ...(i)

$T\sin\theta = P$ ...(ii)

From equation (i) and (ii) we get $P = W\tan\theta$

and $T^2 = P^2 + W^2$

13. (b)

14. (d)

Assertion and Reason

1 (a) Cross product of two vectors is perpendicular to the plane containing both the vectors.

2 (a) $\cos\theta = \frac{(\hat{i}+\hat{j}).(\hat{i})}{|\hat{i}+\hat{j}||\hat{i}|} = \frac{1}{\sqrt{2}}$. Hence $\theta = 45°$.

3 (d) $\frac{\vec{A}\times\vec{B}}{\vec{A}.\vec{B}} = \frac{AB\sin\theta\,\hat{n}}{AB\cos\theta} = \tan\theta\,\hat{n}$

where $\hat{n}$ is unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

However $\frac{|\vec{A}\times\vec{B}|}{\vec{A}.\vec{B}} = \tan\theta$

4 (b) $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 + 2AB\cos\theta$

Hence $\cos\theta = 0$ which gives $\theta = 90°$

Also vector addition is commutative.

Hence $\vec{A}+\vec{B} = \vec{B}+\vec{A}$.

5 (c) $\vec{v} = \vec{\omega}\times\vec{r}$

The expression $\vec{\omega} = \vec{v}\times\vec{r}$ is wrong.

6 (b) For giving a zero resultant, it should be possible to represent the given vectors along the sides of a closed polygon and minimum number of sides of a polygon is three.

7 (a) Since velocities are in opposite direction, therefore $v_{AB} = |\vec{v}_A - \vec{v}_B| = v_A + v_B$.

Which is greater than v_A or v_B

8 (b) Vector addition of two vectors is commutative *i.e.* $\vec{A}+\vec{B} = \vec{B}+\vec{A}$.

9 (a)

10 (c) Cross-product of two vectors is anticommutative.

i.e. $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

11 (b)

12 (e) If a vector quantity has zero magnitude then it is called a null vector. That quantity may have some direction even if its magnitude is zero.

13 (a) Let $\vec{P}$ and $\vec{Q}$ are two vectors in opposite direction, then their sum $\vec{P}+(-\vec{Q}) = \vec{P}-\vec{Q}$

If $\vec{P} = \vec{Q}$ then sum equal to zero.

14 (c) If two vectors are in opposite direction, then they cannot be like vectors.

15 (a) If θ be the angle between two vectors $\vec{A}$ and $\vec{B}$, then their scalar product, $\vec{A}.\vec{B} = AB\cos\theta$

If $\theta = 90°$ then $\vec{A}.\vec{B} = 0$

i.e. if $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product will be zero.

16 (b) We can multiply any vector by any scalar.

For example, in equation $\vec{F} = m\vec{a}$ mass is a scalar quantity, but acceleration is a vector quantity.

17 (c) If two vectors equal in magnitude are in opposite direction, then their sum will be a null vector.

A null vector has direction which is intermediate (or depends on direction of initial vectors) even its magnitude is zero.

18 (b) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta = 0$

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta = 0$

If $\vec{A}$ and $\vec{B}$ are not null vectors then it follows that $\sin\theta$ and $\cos\theta$ both should be zero simultaneously. But it cannot be possible so it is essential that one of the vector must be null vector.

19 (b)

20 (c) The resultant of two vectors of unequal magnitude given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ cannot be zero for any value of θ.

21 (a) $\vec{A}.\vec{B} = \vec{B}.\vec{C} \Rightarrow AB\cos\theta_1 = BC\cos\theta_2$

$\therefore A = C$, only when $\theta_1 = \theta_2$

So when angle between $\vec{A}$ and $\vec{B}$ is equal to angle between $\vec{B}$ and $\vec{C}$ only then $\vec{A}$ equal to $\vec{C}$

22 (c) Since vector addition is commutative, therefore $\vec{A}+\vec{B} = \vec{B}+\vec{A}$.

Vectors

SET Self Evaluation Test - 0

1. $0.4i + 0.8j + ck$ represents a unit vector when c is

(a) – 0.2 (b) $\sqrt{0.2}$

(c) $\sqrt{0.8}$ (d) 0

2. The angles which a vector $\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ makes with X, Y and Z axes respectively are

(a) 60°, 60°, 60° (b) 45°, 45°, 45°

(c) 60°, 60°, 45° (d) 45°, 45°, 60°

3. The value of a unit vector in the direction of vector $A = 5\hat{i} - 12\hat{j}$, is

(a) $\hat{i}$ (b) $\hat{j}$

(c) $(\hat{i} + \hat{j})/13$ (d) $(5\hat{i} - 12\hat{j})/13$

4. Which of the following is independent of the choice of co-ordinate system

(a) $\vec{P} + \vec{Q} + \vec{R}$ (b) $(P_x + Q_x + R_x)\hat{i}$

(c) $P_x\hat{i} + Q_y\hat{j} + R_z\hat{k}$ (d) None of these

5. A car travels 6 km towards north at an angle of 45° to the east and then travels distance of 4 km towards north at an angle of 135° to the east. How far is the point from the starting point. What angle does the straight line joining its initial and final position makes with the east

(a) $\sqrt{50}\ km$ and $\tan^{-1}(5)$

(b) 10 km and $\tan^{-1}(\sqrt{5})$

(c) $\sqrt{52}\ km$ and $\tan^{-1}(5)$

(d) $\sqrt{52}\ km$ and $\tan^{-1}(\sqrt{5})$

6. Given that $\vec{A} + \vec{B} + \vec{C} = 0$ out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(a) 30°, 60°, 90° (b) 45°, 45°, 90°

(c) 45°, 60°, 90° (d) 90°, 135°, 135°

7. Two forces $F_1 = 1\,N$ and $F_2 = 2\,N$ act along the lines $x = 0$ and $y = 0$ respectively. Then the resultant of forces would be

(a) $\hat{i} + 2\hat{j}$ (b) $\hat{i} + \hat{j}$

(c) $3\hat{i} + 2\hat{j}$ (d) $2\hat{i} + \hat{j}$

8. At what angle must the two forces $(x + y)$ and $(x - y)$ act so that the resultant may be $\sqrt{(x^2 + y^2)}$

(a) $\cos^{-1}\left(-\frac{x^2+y^2}{2(x^2-y^2)}\right)$ (b) $\cos^{-1}\left(-\frac{2(x^2-y^2)}{x^2+y^2}\right)$

(c) $\cos^{-1}\left(-\frac{x^2+y^2}{x^2-y^2}\right)$ (d) $\cos^{-1}\left(-\frac{x^2-y^2}{x^2+y^2}\right)$

9. Following forces start acting on a particle at rest at the origin of the co-ordinate system simultaneously

$\vec{F}_1 = -4\hat{i} - 5\hat{j} + 5\hat{k}$, $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$, $\vec{F}_3 = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{F}_4 = 2\hat{i} - 3\hat{j} - 2\hat{k}$ then the particle will move

(a) In $x - y$ plane (b) In $y - z$ plane

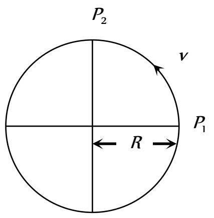
(c) In $x - z$ plane (d) Along x -axis

10. The resultant of $\vec{A} + \vec{B}$ is $\vec{R}_1$. On reversing the vector $\vec{B}$, the resultant becomes $\vec{R}_2$. What is the value of $R_1^2 + R_2^2$

(a) $A^2 + B^2$ (b) $A^2 - B^2$

(c) $2(A^2 + B^2)$ (d) $2(A^2 - B^2)$

11. Figure below shows a body of mass M moving with the uniform speed on a circular path of radius, R. What is the change in acceleration in going from P_1 to P_2



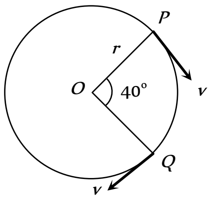
(a) Zero

(b) $v^2/2R$

(c) $2v^2/R$

(d) $\frac{v^2}{R} \times \sqrt{2}$

12. A particle is moving on a circular path of radius r with uniform velocity v. The change in velocity when the particle moves from P to Q is $(\angle POQ = 40°)$



(a) $2v\cos 40°$

(b) $2v\sin 40°$

(c) $2v\sin 20°$

(d) $2v\cos 20°$

13. $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ are two vectors. The angle between them will be

(a) 0° (b) 45°

(c) 60° (d) 90°

14. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

(a) $\frac{3}{\sqrt{13}}$ (b) $\frac{3}{\sqrt{26}}$

(c) $\sqrt{\frac{3}{26}}$ (d) $\sqrt{\frac{3}{13}}$

15. In above example a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ will be

(a) $+\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$ (b) $-\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

(c) Both (a) and (b) (d) None of these

16. Two constant forces $F_1 = 2\hat{i}-3\hat{j}+3\hat{k}$ (N) and $F_2 = \hat{i}+\hat{j}-2\hat{k}$ (N) act on a body and displace it from the position $r_1 = \hat{i}+2\hat{j}-2\hat{k}$ (m) to the position $r_2 = 7\hat{i}+10\hat{j}+5\hat{k}$ (m). What is the work done

(a) 9 J (b) 41 J

(c) -3 J (d) None of these

17. For any two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$, the magnitude of $\vec{C} = \vec{A}+\vec{B}$ is equal to

(a) $\sqrt{A^2+B^2}$ (b) $A+B$

(c) $\sqrt{A^2+B^2+\frac{AB}{\sqrt{2}}}$ (d) $\sqrt{A^2+B^2+\sqrt{2}\times AB}$

18. Which of the following is the unit vector perpendicular to $\vec{A}$ and $\vec{B}$

(a) $\frac{\hat{A}\times\hat{B}}{AB\sin\theta}$ (b) $\frac{\hat{A}\times\hat{B}}{AB\cos\theta}$

(c) $\frac{\vec{A}\times\vec{B}}{AB\sin\theta}$ (d) $\frac{\vec{A}\times\vec{B}}{AB\cos\theta}$

19. Two vectors $P = 2\hat{i}+b\hat{j}+2\hat{k}$ and $Q = \hat{i}+\hat{j}+\hat{k}$ will be parallel if

(a) $b = 0$ (b) $b = 1$

(c) $b = 2$ (d) $b = -4$

20. Which of the following is not true ? If $\vec{A} = 3\hat{i}+4\hat{j}$ and $\vec{B} = 6\hat{i}+8\hat{j}$ where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$

(a) $\vec{A}\times\vec{B} = 0$ (b) $\frac{A}{B} = \frac{1}{2}$

(c) $\vec{A}.\vec{B} = 48$ (d) A = 5

21. The area of the triangle formed by $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ is

(a) 3 sq.unit

(b) $2\sqrt{3}$ sq. unit

(c) $2\sqrt{14}$ sq. unit

(d) $\frac{\sqrt{14}}{2}$ sq. unit

22. Two trains along the same straight rails moving with constant speed 60 *km/hr* and 30 *km/hr* respectively towards each other. If at time $t = 0$, the distance between them is 90 *km*, the time when they collide is

(a) 1 *hr* (b) 2 *hr*

(c) 3 *hr* (d) 4 *hr*

23. A steam boat goes across a lake and comes back (a) On a quite day when the water is still and (b) On a rough day when there is uniform air current so as to help the journey onward and to impede the journey back. If the speed of the launch on both days was same, in which case it will complete the journey in lesser time

(a) Case (a)

(b) Case (b)

(c) Same in both

(d) Nothing can be predicted

24. To a person, going eastward in a car with a velocity of 25 *km/hr*, a train appears to move towards north with a velocity of $25\sqrt{3}$ *km/hr*. The actual velocity of the train will be

(a) 25 *km/hr* (b) 50 *km/hr*

(c) 5 *km/hr* (d) $5\sqrt{3}$ *km/hr*

25. A swimmer can swim in still water with speed υ and the river is flowing with velocity $v/2$. To cross the river in shortest distance, he should swim making angle θ with the upstream. What is the ratio of the time taken to swim across the shortest time to that is swimming across over shortest distance

(a) cos θ (b) sin θ

(c) tan θ (d) cot θ

26. A bus is moving with a velocity 10 m/s on a straight road. A scooterist wishes to overtake the bus in 100 *s*. If the bus is at a distance of 1 *km* from the scooterist, with what velocity should the scooterist chase the bus

(a) 50 *m/s* (b) 40 *m/s*

(c) 30 *m/s* (d) 20 *m/s*

AS Answers and Solutions

(SET -0)

1. (b) $\sqrt{(0.4)^2+(0.8)^2+c^2}=1$

$\Rightarrow 0.16+0.64+c^2=1 \Rightarrow c=\sqrt{0.2}$

2. (c) $\vec{R}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}$

Comparing the given vector with $R=R_x\hat{i}+R_y\hat{j}+R_z\hat{k}$

$R_x=1,\ R_y=1,\ R_z=\sqrt{2}$ and $|\vec{R}|=\sqrt{R_x^2+R_y^2+R_z^2}=2$

$\cos\alpha=\frac{R_x}{R}=\frac{1}{2}\Rightarrow\alpha=60°$

$\cos\beta=\frac{R_y}{R}=\frac{1}{2}\Rightarrow\beta=60°$

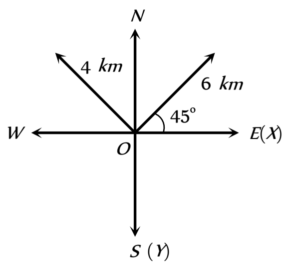
$\cos\gamma=\frac{R_z}{R}=\frac{1}{\sqrt{2}}\Rightarrow\gamma=45°$

3. (d) $\vec{A}=5\hat{i}+12\hat{j}, |\vec{A}|=\sqrt{5^2+(-12)^2}=\sqrt{25+144}=13$

Unit vector $\hat{A}=\frac{\vec{A}}{|\vec{A}|}=\frac{5\hat{i}-12\hat{j}}{13}$

4. (a)

5. (c)



Net movement along x-direction $S_x=(6-4)\cos 45°\ \hat{i}$

$=2\times\frac{1}{\sqrt{2}}=\sqrt{2}\ km$

Net movement along y-direction $S_y=(6+4)\sin 45°\ \hat{j}$

$=10\times\frac{1}{\sqrt{2}}=5\sqrt{2}\ km$

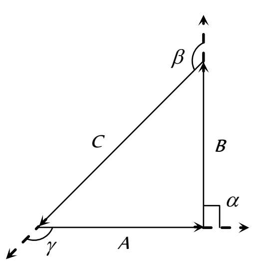
Net movement from starting point

$|\vec{S}|=\sqrt{S_x^2+S_y^2}=\sqrt{(\sqrt{2})^2+(5\sqrt{2})^2}=\sqrt{52}\ km$

Angle which makes with the east direction

$\tan\theta=\frac{\text{Y-component}}{\text{X-component}}=\frac{5\sqrt{2}}{\sqrt{2}}\ \therefore\ \theta=\tan^{-1}(5)$

6. (d)



From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. *i.e.* the triangle should be right angled triangle

Angle between A and B, $\alpha=90°$

Angle between B and C, $\beta=135°$

Angle between A and C, $\gamma=135°$

7. (d) $x=0$ means y-axis $\Rightarrow \vec{F}_1=\hat{j}$

$y=0$ means x-axis $\Rightarrow \vec{F}_2=2\hat{i}$

so resultant $\vec{F}=\vec{F}_1+\vec{F}_2=2\hat{i}+\hat{j}$

8. (a) $R^2=A^2+B^2+2AB\cos\theta$

Substituting, $A=(x+y)$, $B=(x-y)$ and $R=\sqrt{(x^2+y^2)}$

we get $\theta=\cos^{-1}\left(-\frac{(x^2+y^2)}{2(x^2-y^2)}\right)$

9. (b) $\vec{F}_1+\vec{F}_2+\vec{F}_3+\vec{F}_4$

$=(-4\hat{i}+5\hat{i}-3\hat{i}+2\hat{i})+(-5\hat{j}+8\hat{j}+4\hat{j}-3\hat{j})$

$+(5\hat{k}+6\hat{k}-7\hat{k}-2\hat{k})=4\hat{j}+2\hat{k}$

$\therefore$ the particle will move in $y-z$ plane.

10. (c) $\vec{R}_1=\vec{A}+\vec{B},\ \vec{R}_2=\vec{A}-\vec{B}$

$R_1^2+R_2^2=\left(\sqrt{A^2+B^2}\right)^2+\left(\sqrt{A^2+B^2}\right)^2=2\left(A^2+B^2\right)$

11. (d) $\Delta a=2a\sin\left(\frac{\theta}{2}\right)=2a\times\sin 45°=\sqrt{2}a=\sqrt{2}\frac{v^2}{R}$

12. (b) $\Delta v=2v\sin\left(\frac{\theta}{2}\right)=2v\sin 20°$

13. (c) $\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{a_1b_1+a_2b_2+a_3b_3}{|\vec{A}||\vec{B}|}$

$=\frac{2\times 4+4\times 2-4\times 4}{|\vec{A}||\vec{B}|}=0$

$\therefore\theta=\cos^{-1}(0°)\ \Rightarrow\ \theta=90°$

14. (b) $|\vec{A}|=\sqrt{2^2+3^2+(-1)^2}=\sqrt{4+9+1}=\sqrt{14}$

$|\vec{B}|=\sqrt{(-1)^2+3^2+4^2}=\sqrt{1+9+16}=\sqrt{26}$

$\vec{A}.\vec{B}=2(-1)+3\times 3+(-1)(4)=3$

The projection of $\vec{A}$ on $\vec{B}=\frac{\vec{A}.\vec{B}}{|\vec{B}|}=\frac{3}{\sqrt{26}}$

15. (c) $\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\frac{8\hat{i}-8\hat{j}-8\hat{k}}{8\sqrt{3}}=\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$

they are $\hat{n}=\pm\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

16. (a) $W = \bar{F}(\bar{r}_2 - \bar{r}_1)$

$= (3\hat{i} - 2\hat{j} + \hat{k})(6\hat{i} + 8\hat{j} + 7\hat{k}) = 18 - 16 + 7 = 9\,J$

17. (d) $AB\cos\theta = AB\sin\theta \Rightarrow \tan\theta = 1 \therefore \theta = 45°$

$\therefore |\bar{C}| = \sqrt{A^2 + B^2 + 2AB\cos 45°} = \sqrt{A^2 + B^2 + \sqrt{2}AB}$

18. (c) Vector perpendicular to A and B, $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

$\therefore$ Unit vector perpendicular to A and B

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A}| \times |\vec{B}|\,\sin\theta}$$

19. (c) P and Q will be parallel if $\frac{2}{1} = \frac{b}{1} = \frac{2}{1} \therefore b = 2$

20. (b) $|\vec{A}| = 5$, $|\vec{B}| = 10 \Rightarrow \frac{A}{B} = \frac{1}{2}$

21. (d) $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

Area of the triangle $= \frac{1}{2}\left(\vec{A} \times \vec{B}\right)$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left|2\hat{i} - 3\hat{j} + \hat{k}\right| = \frac{1}{2}\sqrt{4 + 9 + 1}$$

$$= \frac{\sqrt{14}}{2}\,sq.unit$$

22. (a) The relative velocity $v_{rel.} = 60 - (-30) = 90\ km/hr$.

Distance between the train $s_{rel.} = 90\ km$,

$\therefore$ Time when they collide $= \frac{s_{rel.}}{v_{rel.}} = \frac{90}{90} = 1\,hr$.

23. (b) If the breadth of the lake is l and velocity of boat is v_b. Time in going and coming back on a quite day

$$t_Q = \frac{l}{v_b} + \frac{l}{v_b} = \frac{2l}{v_b} \quad .....(i)$$

Now if v_a is the velocity of air- current then time taken in going across the lake,

$$t_1 = \frac{l}{v_b + v_a} \quad \text{[As current helps the motion]}$$

and time taken in coming back $t_2 = \frac{l}{v_b - v_a}$

[As current opposes the motion]

So $t_R = t_1 + t_2 = \frac{2l}{v_b[1 - (v_a/v_b)^2]}$ .....(ii)

From equation (i) and (ii)

$$\frac{t_R}{t_Q} = \frac{1}{[1 - (v_a/v_b)^2]} > 1 \ \left[\text{as } 1 - \frac{v_a^2}{v_b^2} < 1\right] \ \text{i.e. } t_R > t_Q$$

i.e. time taken to complete the journey on quite day is lesser than that on rough day.

24. (a) $v_T = \sqrt{v_{TC}^2 + v_C^2} = \sqrt{(25\sqrt{3})^2 + (25)^2}$

$= \sqrt{1875 + 625} = \sqrt{2500} = 25\ km/hr$

25. (b)

26. (d) Let the velocity of the scooterist $= v$

Relative velocity of scooterist with respect to bus $= (v - 10)$

$\Rightarrow S = (v - 10) \times 100 \Rightarrow 1000 = (v - 10) \times 100$

$\therefore v = 10 + 10 = 20\,m/s$

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